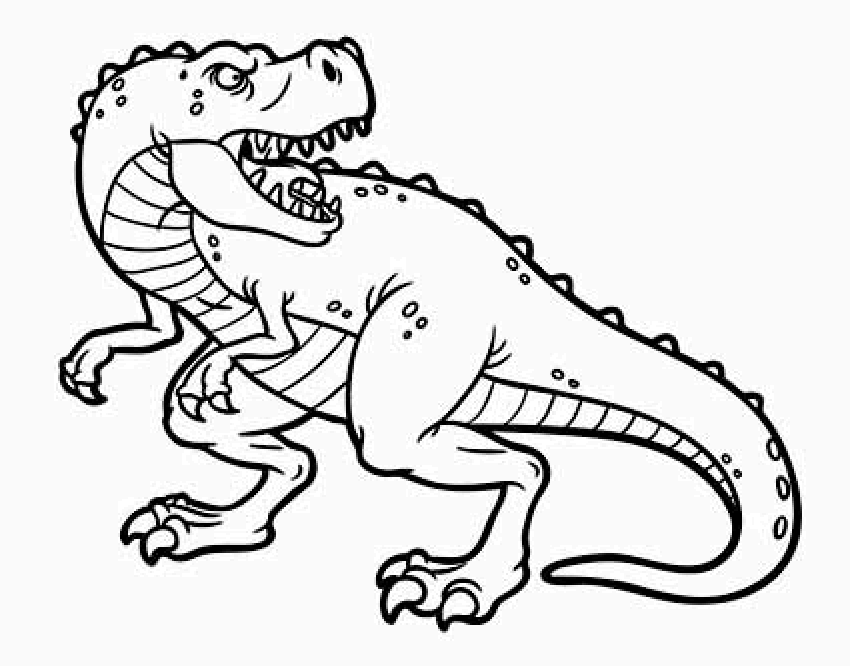
Prep [1]

Geometry - Second Term

Unit [3] - Part [1]



Mr. Mahmoud Esmaiel

01006487539 - 01110882717

الاســم ....

Prep [1] - Second Term - Unit [3] : Geometry And Measurement

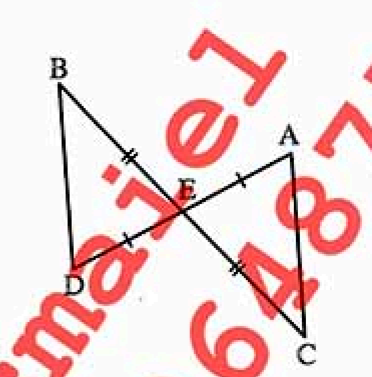
Lesson [1] : Deductive Proof

Example [1]

In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE and BE = CE

Prove that : Δ AEC $\equiv \Delta$ DEB



Solutions

Given $\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE , BE = CE

R.T.P. Δ AEC $\equiv \Delta$ DEB

Proof $\because \overline{AD} \cap \overline{BC} = \{E\}$ $\therefore$ m ($\angle$ AEC) = m ($\angle$ DEB) (V.O.A)

$\therefore$ In $\Delta\Delta$ AEC and DEB :

$\begin{cases} AE = DE \text{ (given)} \\ CE = BE \text{ (given)} \\ m(\angle AEC) = m(\angle DEB) \text{ (by proof)} \end{cases}$

$\therefore \Delta$ AEC $\equiv \Delta$ DEB (Q.E.D.)

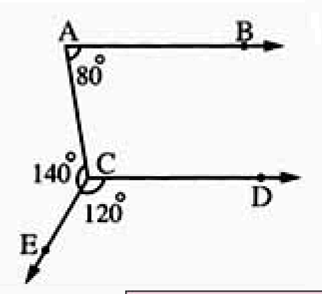
Example [2]

In the opposite figure :

m ($\angle$ BAC) = 80° , m ($\angle$ DCE) = 120°

and m ($\angle$ ACE) = 140°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{CD}$



Solutions

Given m ($\angle$ BAC) = 80° , m ($\angle$ DCE) = 120° ,

m ($\angle$ ACE) = 140°

R.T.P. $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Proof $\because$ m ($\angle$ DCA) + m ($\angle$ DCE) + m ($\angle$ ACE) = 360°

(accumulative angles at C)

$\therefore$ m ($\angle$ DCA) = 360° – (120° + 140°) = 100°

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DCA) = 80° + 100° = 180°

And they are interior angles in the same side of the transversal $\overleftrightarrow{AC}$

$\therefore$ $\overrightarrow{AB}$ // $\overrightarrow{CD}$ (Q.E.D.)

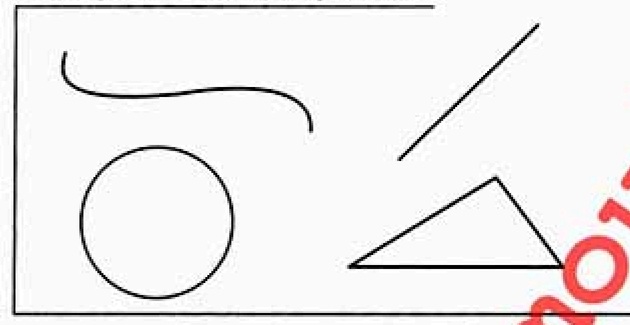
Lesson [2] : Part [1] : The Polygon

Before studying polygons we will study the types of the line as follows.

The simple line

It is the line that does not cut itself.

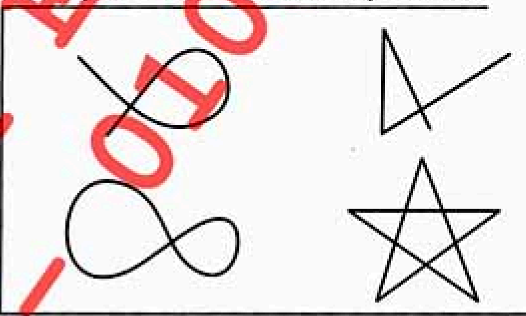
Examples for the simple line :



The non-simple line

It is the line that cuts itself once or more.

Examples for the non-simple line :



The closed line

It is the line that ends where it starts at the same point. It may be simple or non-simple.

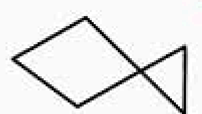
Examples for the closed line :



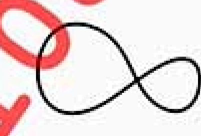
The closed simple line.



The closed simple line.



The closed non-simple line.

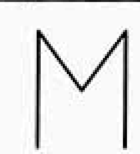


The closed non-simple line.

The open line

It is the line whose starting point is not the end point. It may be simple or non-simple.

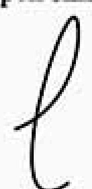
Examples for the open line :



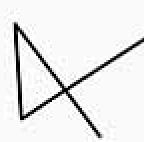
The open simple line.



The open simple line.



The open non-simple line.

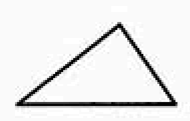


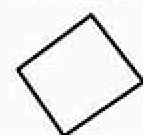
The open non-simple line.

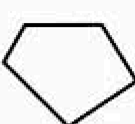
The Polygon :

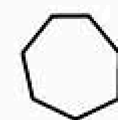
It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.

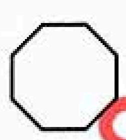
Examples for some polygons :


Triangle
(3 sides)


Quadrilateral
(4 sides)

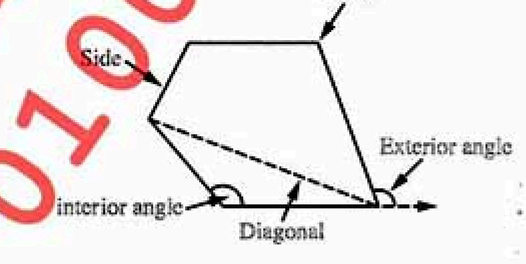

Pentagon
(5 sides)


Heptagon
(7 sides)


Octagon
(8 sides)

Remarks

1. Each line segment of the line segments forming the polygon is called a side.
2. Each point resulted from intersecting of two adjacent sides of the polygon is called a vertex.
3. The sum of the side lengths of the polygon is called the perimeter of the polygon.
4. Each line segment joining two non-adjacent vertices of the polygon is called a diagonal of the polygon.
5. The included angle between two adjacent sides of the polygon is called an interior angle.

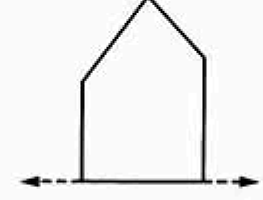


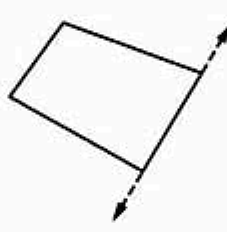
6. The included angle between a side of the polygon and the extension of its adjacent side is called an exterior angle.

Convex Polygon And Concave Polygon :

In the convex polygon :

If a straight line is drawn to pass through any two consecutive vertices, then the remained vertices lie on one side of this straight line.



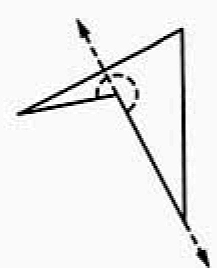


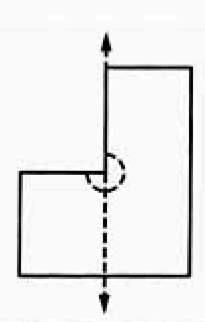
Notice that :

Any interior angle of the convex polygon has measure less than 180°

In the concave polygon :

There are straight lines (one at least) passing through two consecutive vertices and the remained vertices lie on two different sides of the straight line.





Notice that :

There is at least one interior angle of concave polygon of measure more than 180° (reflex angle).

The Sum Of Measures Of The Interior Angles Of The Polygon :

∴ The sum of measures of the interior angles of a polygon of n sides equals $(n - 2) \times 180°$

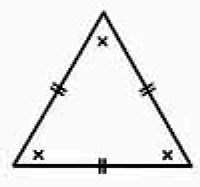
The sum of measures of the exterior angles of a convex polygon of n sides = 360°
(taking into account one exterior angle at each vertex)

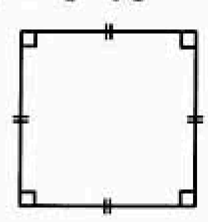
The Regular Polygon :

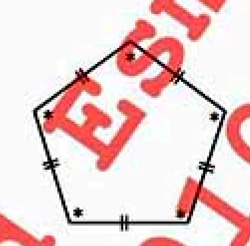
The polygon is regular if :

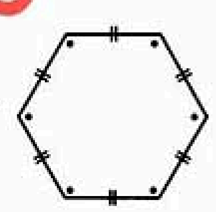
1. All its sides are equal in length.
2. All its angles are equal in measure.

As examples for the regular polygons :


Equilateral triangle


Square


Regular pentagon


Regular hexagon

The Measure Of The Interior Angle Of A Regular Polygon :

∴ The measure of each interior angle of the regular polygon of n-sides = $\frac{(n-2) \times 180°}{n}$

For example:

- The measure of each interior angle of the equilateral triangle = $\frac{(3-2) \times 180°}{3} = 60°$
- The measure of each interior angle of the square = $\frac{(4-2) \times 180°}{4} = 90°$
- The measure of each interior angle of the regular pentagon = $\frac{(5-2) \times 180°}{5} = 108°$
- The measure of each interior angle of the regular hexagon = $\frac{(6-2) \times 180°}{6} = 120°$

Notice that :

The number of the polygon sides = The number of its vertices
= The number of its interior angles = The number of its exterior angles

Remarks

The number of sides of the regular polygon in which the measure of one of its interior angles is $x° = \frac{360°}{180° - x}$

Number of diagonals = n (n – 3) ÷ 2

Exercises

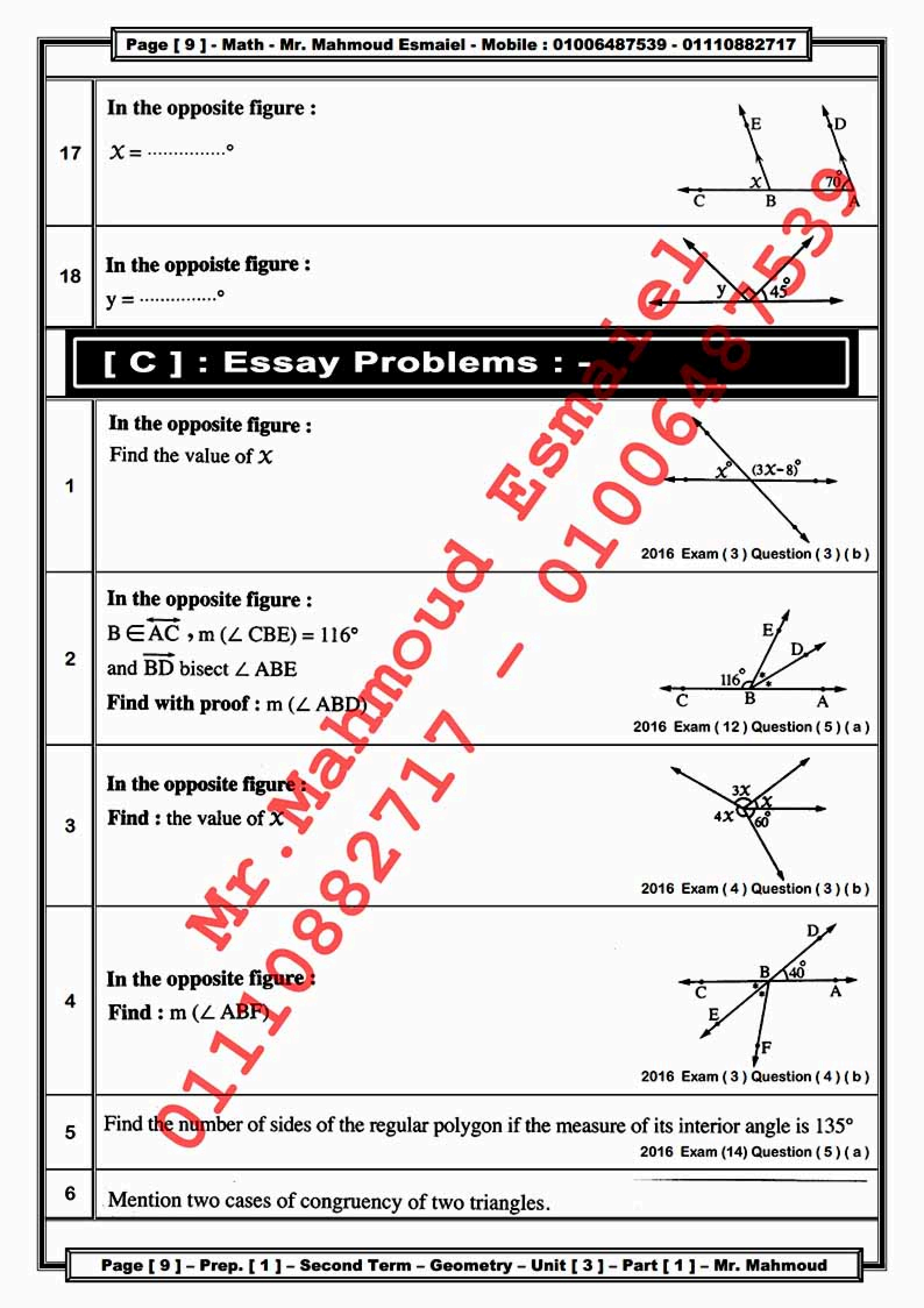
[A] : Choose The Correct Answer : -

1	The measure of the right angle = ... ° (a) 180 (b) 90 (c) 120 (d) 0
2	The angle whose measure 90° is .... angle. (a) acute (b) right (c) obtuse (d) straight
3	The sum of the measures of the accumulative angles at a point = ... (a) 90° (b) 180° (c) 270° (d) 360°
4	If Δ ABC $\equiv$ Δ XYZ , then AB = ... (a) XY (b) YZ (c) XZ (d) BC
5	The angle with measure 70° complement angle with measure ....° (a) 70 (b) 110 (c) 290 (d) 20
6	The acute angle supplements .... angle. (a) acute (b) right (c) obtuse (d) straight
7	The area of the circle = ... (a) π r (b) πr^2 (c) 2 π r (d) $2 \pi r^2$
8	The two bisectors of two adjacent supplementary angles included an angle of measure ....° (a) 180 (b) 45 (c) 90 (d) 0
9	The perpendicular to one of two parallel lines is .... to the other. (a) parallel (b) equal (c) congruent (d) perpendicular
10	The edge length of a cube whose total area is 600 cm^2. is .... cm. (a) 10 (b) 100 (c) 300 (d) 90
11	The hexagon has .... sides. (a) 5 (b) 6 (c) 7 (d) 8
12	The pentagon has .... sides. (a) 3 (b) 4 (c) 5 (d) 6
13	The sum of the measures of the exterior angles of a polygon of n sides is .... (a) (n – 2) (b) (n – 2) × 180° (c) 360° (d) $\frac{(n-2) \times 180°}{n}$
14	The sum of the measures of the interior angles of a triangle = ...° (a) 90 (b) 360 (c) 180 (d) 540

15	The sum of the measures of the interior angles of a pentagon is .... (a) 360° (b) 450° (c) 720° (d) 540°
16	The area of the shaded part = ... the total area of the shape. (a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$
17	In all the following shapes m (∠ X) = 60° except the shape .... X, 120° (a) X, 40° (b) X (c) X, 120° (d)
18	The sum of measures of the exterior angles of the hexagon = ... (a) 720 ° (b) 120 ° (c) 180 ° (d) 360 °
19	The measure of the exterior angle of the equilateral triangle = ... (a) 60° (b) 90° (c) 30° (d) 120°
20	If the number of sides of a regular polygon is 5 and if the measure of each interior angle is (x°) , then x = ... (a) 90° (b) 108° (c) 120° (d) 180°
21	The measure of the interior angle of a regular pentagon = ... (a) 900° (b) 180° (c) 540° (d) 108°
22	The measure of each angle of the regular hexagon is .... (a) 90° (b) 180° (c) 120° (d) 144°
23	The measure of the interior angle of a regular polygon of 18 sides equals .... (a) 130° (b) 140° (c) 150° (d) 160°
24	How many sides has a regular polygon if the measure of each interior angle of it is 120° ? (a) 5 (b) 6 (c) 7 (d) 8
25	If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is .... (a) 6 (b) 4 (c) 7 (d) 8
26	The number of diagonals of a quadrilateral is .... (a) 4 (b) 3 (c) 2 (d) 0

[B] : Complete the Following : -

1	The angle of measure 180° its type is ……………
2	The measure of the straight angle equals ……………°
3	The measure of the right angle = ……………°
4	The sum of the measures of the accumulative angles at a point is ……………°
5	The two vertically opposite angles are ……………
6	If two straight lines intersect , then the measures of each two vertically opposite angles are ……………
7	Every two vertically opposite angles are …………… in measure.
8	Each two opposite angles in a parallelogram are ……………
9	If two straight lines intersect , then the sum of measures of any two adjacent angles is ……………
10	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ……………
11	The opposite figure represents 3 squares each of side length 1 cm. , the perimeter of the figure = ……………
12	A circle its radius length 10 cm. , then its circumference = …………… (Consider $\pi = 3.14$)
13	The sum of the measures of the exterior angles of the convex polygon = ……………
14	The sum of the measures of the angles of the quadrilateral equals ……………
15	The measure of each interior angle of the regular hexagon is ……………°
16	The measure of each interior angle of the regular pentagon = ……………



17 **In the opposite figure :**

x = ...°

18 **In the oppoiste figure :**

y = ...°

[C] : Essay Problems : -

1 **In the opposite figure :**

Find the value of x

2016 Exam (3) Question (3) (b)

2 **In the opposite figure :**

B ∈ $\overleftrightarrow{AC}$, m (∠ CBE) = 116°

and $\overrightarrow{BD}$ bisect ∠ ABE

Find with proof : m (∠ ABD)

2016 Exam (12) Question (5) (a)

3 **In the opposite figure :**

Find : the value of x

2016 Exam (4) Question (3) (b)

4 **In the opposite figure :**

Find : m (∠ ABF)

2016 Exam (3) Question (4) (b)

5 Find the number of sides of the regular polygon if the measure of its interior angle is 135°

2016 Exam (14) Question (5) (a)

6 Mention two cases of congruency of two triangles.

2017 Exam (12) Question (5) (a)

7

Using the geometric tools , draw the angle ABC of measure 140 , then bisect it. (don't remove arcs).

2017 Exam (12) Question (4) (b)

8

In the opposite figure :

m (∠ A) = 80° , m (∠ D) = 120° ,

m (∠ CBE) = 130° and B ∈ $\overrightarrow{AE}$

Find with proof : m (∠ C)

2016 Exam (6) Question (3) (a)

9

In the opposite figure :

F ∈ $\overrightarrow{YZ}$, m (∠ L) = 70°

, m (∠ Y) = 90° and m (∠ LZF) = 120°

Find : m (∠ X)

2018 Exam (13) Question (3) (b)

10

In the opposite figure :

$\overleftrightarrow{AD}$ // $\overrightarrow{BC}$, $\overrightarrow{CF}$ bisects ∠ DCE

, m (∠ ABC) = 55° , m (∠ ADC) = 110°

Prove that : $\overline{AB}$ // $\overrightarrow{CF}$

2017 Exam (11) Question (5) (b)

11

In the opposite figure :

ABCD is a quadrilateral

in which : m (∠ A) = 90°

Find : the value of x

2018 Exam (11) Question (3) (a)

12

In the opposite figure :

$\overrightarrow{AB}$ // $\overleftrightarrow{CD}$, m (∠ A) = 50° ,

∠ ACE is right angle ,

and m (∠ E) = 40°

Prove that : $\overrightarrow{AB}$ // $\overrightarrow{EF}$

2016 Exam (5) Question (5) (b)

Homework

[A] : Choose The Correct Answer : -

1 The edge length of a cube whose total area is 600 cm^2. is .... cm.

(a) 10 (b) 100 (c) 300 (d) 90

2 If the number of sides of a regular polygon is 5 and if the measure of each interior angle is ($x°$) , then x = ...

(a) 90° (b) 108° (c) 120° (d) 180°

3 The perpendicular to one of two parallel lines is .... to the other.

(a) parallel (b) equal (c) congruent (d) perpendicular

4 The measure of the exterior angle of the equilateral triangle = ...

(a) 60° (b) 90° (c) 30° (d) 120°

5 The two bisectors of two adjacent supplementary angles included an angle of measure ....°

(a) 180 (b) 45 (c) 90 (d) 0

6 The sum of measures of the exterior angles of the hexagon = ...

(a) 720 ° (b) 120 ° (c) 180 ° (d) 360 °

7 The area of the circle = ...

(a) π r (b) πr^2 (c) 2 π r (d) $2 \pi r^2$

8 In all the following shapes m (∠ X) = 60° except the shape ....

X, 120° (a) X, 40° (b) X (c) X, 120° (d)

9 The number of diagonals of a quadrilateral is ....

(a) 4 (b) 3 (c) 2 (d) 0

10 The angle with measure 70° complement angle with measure ....°

(a) 70 (b) 110 (c) 290 (d) 20

11 If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is ....

(a) 6 (b) 4 (c) 7 (d) 8

12 If $\Delta ABC \equiv \Delta XYZ$, then AB = ……………

(a) XY (b) YZ (c) XZ (d) BC

13 The acute angle supplements …………… angle.

(a) acute (b) right (c) obtuse (d) straight

14 The area of the shaded part = …………… the total area of the shape.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

15 The sum of the measures of the interior angles of a triangle = ……………°

(a) 90 (b) 360 (c) 180 (d) 540

16 How many sides has a regular polygon if the measure of each interior angle of it is 120° ?

(a) 5 (b) 6 (c) 7 (d) 8

17 The sum of the measures of the accumulative angles at a point = ……………

(a) 90° (b) 180° (c) 270° (d) 360°

18 The sum of the measures of the exterior angles of a polygon of n sides is ……………

(a) $(n - 2)$ (b) $(n - 2) \times 180°$ (c) 360° (d) $\frac{(n-2) \times 180°}{n}$

19 The measure of the interior angle of a regular polygon of 18 sides equals ……………

(a) 130° (b) 140° (c) 150° (d) 160°

20 The angle whose measure 90° is …………… angle.

(a) acute (b) right (c) obtuse (d) straight

21 The pentagon has …………… sides.

(a) 3 (b) 4 (c) 5 (d) 6

22 The measure of each angle of the regular hexagon is ……………

(a) 90° (b) 180° (c) 120° (d) 144°

23 The measure of the right angle = …………… °

(a) 180 (b) 90 (c) 120 (d) 0

24 The sum of the measures of the interior angles of a pentagon is ……………

(a) 360° (b) 450° (c) 720° (d) 540°

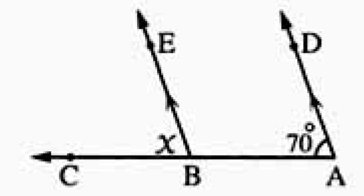
25 The measure of the interior angle of a regular pentagon = ……………

(a) 900° (b) 180° (c) 540° (d) 108°

26 The hexagon has …………… sides.

(a) 5 (b) 6 (c) 7 (d) 8

[B] : Complete the Following : -

No.	Statement
1	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ……………
2	If two straight lines intersect , then the sum of measures of any two adjacent angles is ……………
3	Each two opposite angles in a parallelogram are ……………
4	**In the oppoiste figure :** y = ……………°
5	 Every two vertically opposite angles are …………… in measure.
6	 **In the opposite figure :** x = ……………° 
7	If two straight lines intersect , then the measures of each two vertically opposite angles are ……………
8	The measure of each interior angle of the regular pentagon = ……………
9	The two vertically opposite angles are ……………
10	The angle of measure 180° its type is ……………
11	The measure of each interior angle of the regular hexagon is ……………°
12	The sum of the measures of the accumulative angles at a point is ……………°
13	The sum of the measures of the angles of the quadrilateral equals ……………
14	The measure of the right angle = ……………°
15	The sum of the measures of the exterior angles of the convex polygon = ……………

16	The measure of the straight angle equals ....°
17	A circle its radius length 10 cm. , then its circumference = ... (Consider $\pi = 3.14$)
18	The opposite figure represents 3 squares each of side length 1 cm. , the perimeter of the figure = ...

[C] : Essay Problems : -

1	**In the opposite figure :** $\overline{AD} \parallel \overrightarrow{BC}$, $\overrightarrow{CF}$ bisects $\angle$ DCE , m ($\angle$ ABC) = 55° , m ($\angle$ ADC) = 110° **Prove that :** $\overline{AB} \parallel \overrightarrow{CF}$ 2017 Exam (11) Question (5) (b)
2	**In the opposite figure :** F $\in \overrightarrow{YZ}$, m ($\angle$ L) = 70° , m ($\angle$ Y) = 90° and m ($\angle$ LZF) = 120° **Find :** m ($\angle$ X) 2018 Exam (13) Question (3) (b)
3	**In the opposite figure :** m ($\angle$ A) = 80° , m ($\angle$ D) = 120° , m ($\angle$ CBE) = 130° and B $\in \overline{AE}$ **Find with proof :** m ($\angle$ C) 2016 Exam (6) Question (3) (a)
4	Using the geometric tools , draw the angle ABC of measure 140 , then bisect it. (don't remove arcs). 2017 Exam (12) Question (4) (b)
5	**In the opposite figure :** Find the value of x 2016 Exam (3) Question (3) (b)

6	Find the number of sides of the regular polygon if the measure of its interior angle is 135° 2016 Exam (14) Question (5) (a)
7	Mention two cases of congruency of two triangles. 2017 Exam (12) Question (5) (a)
8	**In the opposite figure :** **Find :** m (∠ ABF) D, B, 40°, C, A, E, F 2016 Exam (3) Question (4) (b)
9	**In the opposite figure :** **Find :** the value of x $3x$, x, $4x$, 60° 2016 Exam (4) Question (3) (b)
10	**In the opposite figure :** $B \in \overleftrightarrow{AC}$, m (∠ CBE) = 116° and $\overrightarrow{BD}$ bisect ∠ ABE **Find with proof :** m (∠ ABD) E, D, 116°, C, B, A 2016 Exam (12) Question (5) (a)
11	**In the opposite figure :** $\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ A) = 50° , ∠ ACE is right angle , and m (∠ E) = 40° **Prove that :** $\overrightarrow{AB}$ // $\overrightarrow{EF}$ A, B, 50°, D, C, 40°, E, F 2016 Exam (5) Question (5) (b)
12	**In the opposite figure :** ABCD is a quadrilateral in which : m (∠ A) = 90° **Find :** the value of x A, $3x$, B, D, $2x$, $5x$, C 2018 Exam (11) Question (3) (a)

Prep [1]

Geometry - Second Term

Unit [3] - Part [2]



Mr. Mahmoud Esmaiel

01006487539 - 01110882717

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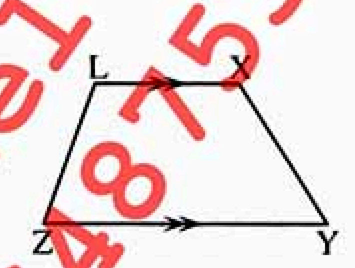
Lesson [2] : Part [2] : The Polygon

Trapezium – Parallelogram – Rectangle – Rhombus – Square

Trapezium :

A quadrilateral in which only two sides are parallel is called a trapezium , as shown in the opposite figure in which :

$\overline{XL} \parallel \overline{YZ}$



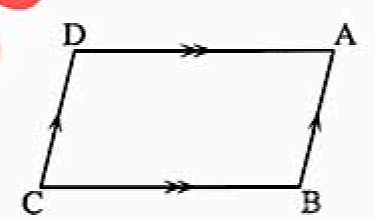
Definition

Parallelogram is a quadrilateral , in which each two opposite sides are parallel.

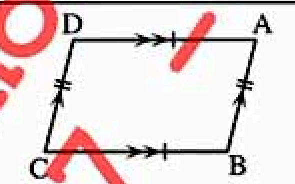
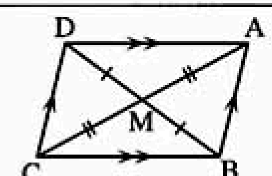
In the opposite figure

ABCD is a parallelogram because

$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$



Properties of parallelogram :

Property	Figure	Result
1 Each two opposite sides are equal in length.		• AB = DC • AD = BC
2 Each two opposite angles are equal in measure.		• m (∠ A) = m (∠ C) • m (∠ B) = m (∠ D)
3 The sum of measures of each two consecutive angles is 180°		• m (∠ A) + m (∠ B) = 180° • m (∠ B) + m (∠ C) = 180° • m (∠ C) + m (∠ D) = 180° • m (∠ D) + m (∠ A) = 180°
4 The two diagonals bisect each other.		• AM = CM • BM = DM

Remark [1]

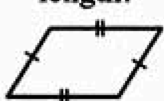
The perimeter of the parallelogram = The sum of two consecutive sides × 2

When does a quadrilateral represent a parallelogram ?

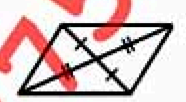
A quadrilateral represents a parallelogram if one of the following conditions satisfies

- Each two opposite sides are parallel.
- Each two opposite sides are equal in length.
- Two opposite sides are parallel and equal in length.
- Each two opposite angles are equal in measure.
- The two diagonals bisect each other.









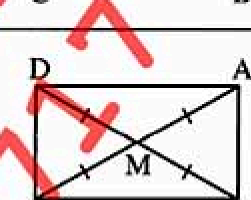
Rectangle :

Rectangle is a parallelogram with a right angle.



Properties Of Rectangle :

The rectangle has the same properties of the parallelogram and some additional properties as the following :

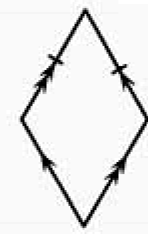
1 The four angles of the rectangle are all equal in measure and the measure of each is 90°	D A C B	m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°
2 The two diagonals of the rectangle are equal in length.		AC = BD and as the two diagonals bisect each other , then AM = BM = CM = DM

Remark [2]

The perimeter of the rectangle = (length + width) × 2

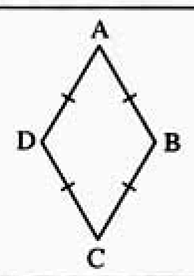
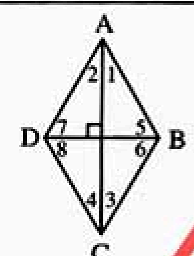
Rhombus :

Rhombus is a parallelogram in which two adjacent sides are equal in length.



Properties Of Rhombus :

The rhombus has the same properties of the parallelogram and some additional properties as the following :

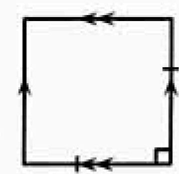
1 The four sides of the rhombus are all equal in length.		• AB = BC = CD = DA
2 The two diagonals of the rhombus are perpendicular and bisect each of its interior angles.		• $\overline{AC} \perp \overline{BD}$ • m (∠ 1) = m (∠ 2) = m (∠ 3) = m (∠ 4) • m (∠ 5) = m (∠ 6) = m (∠ 7) = m (∠ 8)

Remark [3]

The perimeter of the rhombus = the length of one side × 4

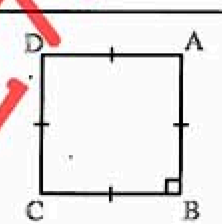
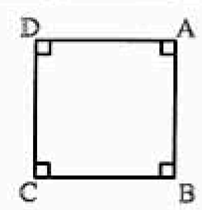
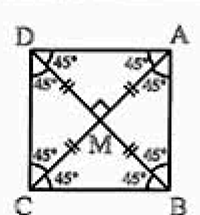
Square :

Square is a parallelogram with a right angle and two adjacent sides are equal in length.



Properties Of Square :

The square has the same properties of the parallelogram and some additional properties as the following :

1 Its four sides are all equal in length.		• AB = BC = CD = DA
2 Its four angles are all equal in measure and each of them is of measure 90°		• m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°
3 Its two diagonals are equal in length , perpendicular and each diagonal bisects the two vertices angles which this diagonal joins.		• AC = BD and hence AM = BM = CM = DM • $\overline{AC} \perp \overline{BD}$

Remark [4]

The perimeter of the square = the length of one side $\times 4$

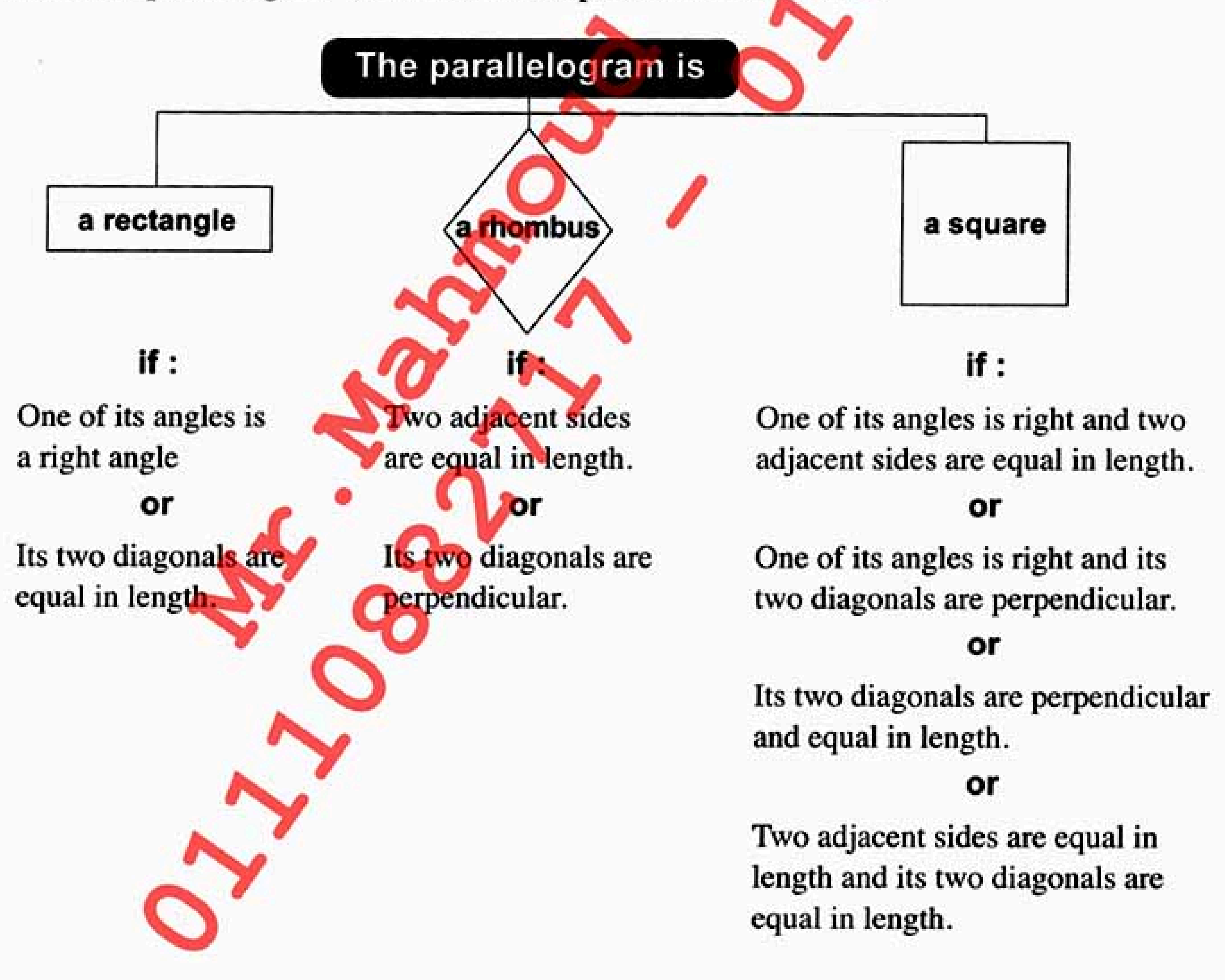
Notice That : -

We can also define the square as follows :

1. A square is a rectangle with two adjacent sides equal in length.
2. A square is a rectangle with two perpendicular diagonals.
3. A square is a rhombus with a right angle.
4. A square is a rhombus with two diagnals equal in length.

Notice That : -

To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then :



a rectangle if :

One of its angles is a right angle

or

Its two diagonals are equal in length.

a rhombus if :

Two adjacent sides are equal in length.

or

Its two diagonals are perpendicular.

a square if :

One of its angles is right and two adjacent sides are equal in length.

or

One of its angles is right and its two diagonals are perpendicular.

or

Its two diagonals are perpendicular and equal in length.

or

Two adjacent sides are equal in length and its two diagonals are equal in length.

Examples :

1 Calculate the sum of the measures of the interior angles of a hexagon.

2014 Exam (12) Question (3) (a)

2 Find the measure of each interior angle of regular hexagon.

2014 Exam (9) Question (4) (a)

3 How many sides does a regular polygon have if the measure of each interior angle of it is 120° ?

2014 Exam (6) Question (3) (b)

4 Find the number of sides of a regular polygon if the measure of one of its exterior angle is 45°

2015 Exam (5) Question (4) (a)

5 **In the opposite figure :**

$\overline{EF} \perp \overrightarrow{FB}$, $\overrightarrow{DA} \cap \overrightarrow{FB} = \{C\}$,

m (∠ ACB) = 66° , m (∠ E) = 104°

Find : m (∠ D)

2015 Exam (15) Question (4) (b)

6 **In the opposite figure :**

$\overline{AD} \parallel \overrightarrow{BC}$, $\overrightarrow{CF}$ bisects ∠ DCE

, m (∠ ABC) = 55° , m (∠ ADC) = 110°

Prove that : $\overline{AB} \parallel \overrightarrow{CF}$

2017 Exam (12) Question (3) (a)

7 **In the opposite figure :**

$\overline{DC} \parallel \overline{AB}$, $E \in \overleftrightarrow{AB}$, m (∠ CBE) = 53° , m (∠ D) = 127°

Prove that : $\overline{AD} \parallel \overline{BC}$

2015 Exam (11) Question (3) (a)

8 **In the opposite figure :**

ABCDE is a pentagon in which $\overline{AE} \parallel \overline{BC}$,

m (∠ A) = 80° , m (∠ C) = 140° , m (∠ E) = 120°

Find : ① m (∠ B)

② m (∠ D)

2015 Exam (4) Question (4) (a)

9

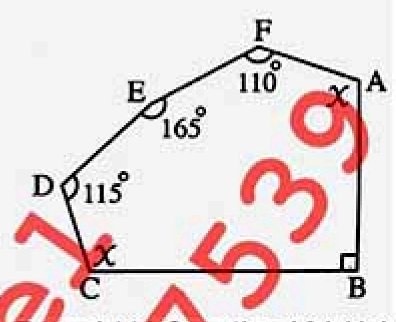
In the opposite figure :

ABCDEF is a hexagon , m (∠ B) = 90° ,

m (∠ F) = 110° , m (∠ E) = 165° , m (∠ D) = 115° ,

m (∠ FAB) = m (∠ DCB) = x

Find : the value of x



2014 Exam (11) Question (3) (b)

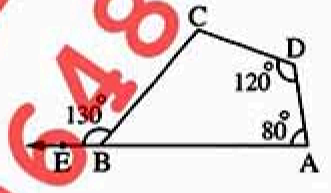
10

In the opposite figure :

m (∠ A) = 80° , m (∠ D) = 120°

, m (∠ CBE) = 130°

Find : m (∠ C)



Model Exam (5) Question (5) (a)

Solutions

1	The sum = (6 – 2) × 180° = 720°
2	The measure of each interior angle = $\frac{(6-2)\times 180°}{6}$ = 120°
3	The number of sides = $\frac{360°}{180° - 120°}$ = 6 sides
4	The number of sides = $\frac{360°}{45°}$ = 8 sides
5	∵ $\overrightarrow{DA} \cap \overrightarrow{FB}$ = {C} ∴ m (∠ DCF) = m (∠ ACB) = 66° (V.O.A.) From quadrilateral CDEF : ∴ m (∠ D) = 360° – (104° + 66° + 90°) = 100° (The req.)
6	∵ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{DC}$ is a transversal ∴ m (∠ DCE) = m (∠ ADC) = 110° (alternate angles) , ∵ $\overrightarrow{CF}$ bisects ∠ DCE ∴ m (∠ FCE) = $\frac{110°}{2}$ = 55° , ∵ m (∠ B) = m (∠ FCE) = 55° and they are corresponding angles. ∴ $\overline{AB}$ // $\overline{CF}$ (Q.E.D.)

7	∵ $\overline{DC}$ // $\overline{AB}$, $\overleftrightarrow{AD}$ is a transversal ∴ m (∠ A) + m (∠ D) = 180° (Two interior angles in the same side of the transversal). ∴ m (∠ A) = 180° – 127° = 53° ∴ m (∠ A) = m (∠ CBE) = 53° and they are corresponding angles. ∴ $\overline{AD}$ // $\overline{BC}$ (Q.E.D.)
8	∵ $\overline{AE}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal ∴ m (∠ A) + m (∠ B) = 180° Two interior angles in the same side of the transversal. ∴ m (∠ B) = 180° – 80° = 100° (First req.) From pantagon ABCDE : ∴ m (∠ D) = 540° – (120° + 80° + 100° + 140°) = 100° (Second req.)
9	∵ 2 x = 720° – (110° + 90° + 165° + 115°) = 240° ∴ x = 240° ÷ 2 = 120° (The req.)
10	∵ B ∈ $\overrightarrow{AE}$ ∴ m (∠ ABC) = 180° – 130° = 50° ∴ From the quadrilateral ABCD : m (∠ C) = 360° – (50° + 80° + 120°) = 360° – 250° = 110° (The req.)

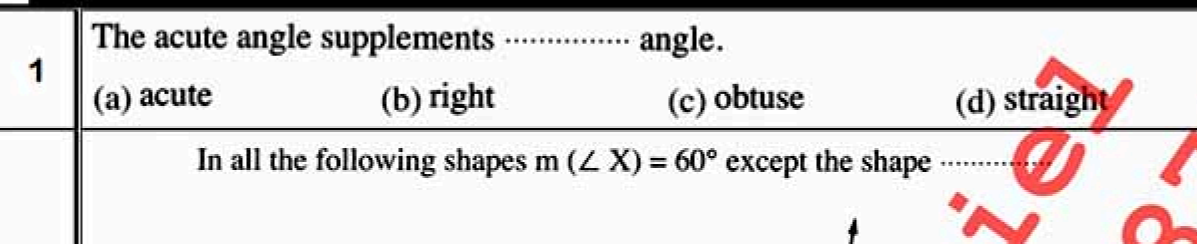
Exercises

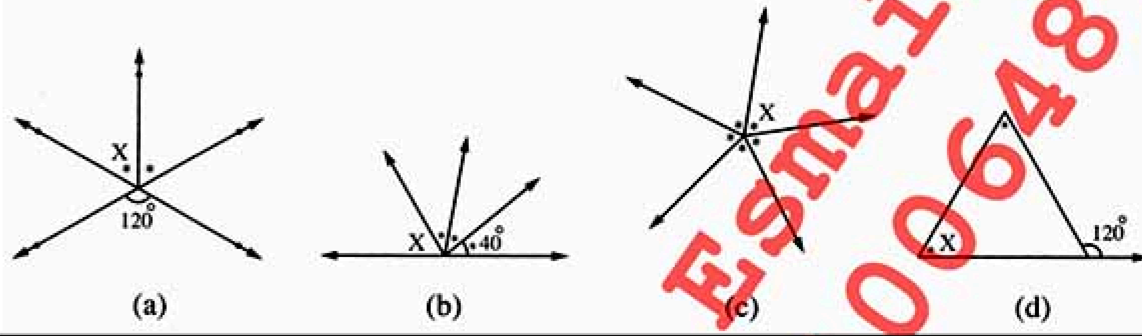
[A] : Choose The Correct Answer : -

1 The acute angle supplements .... angle.

(a) acute (b) right (c) obtuse (d) straight

2 In all the following shapes m (∠ X) = 60° except the shape ....



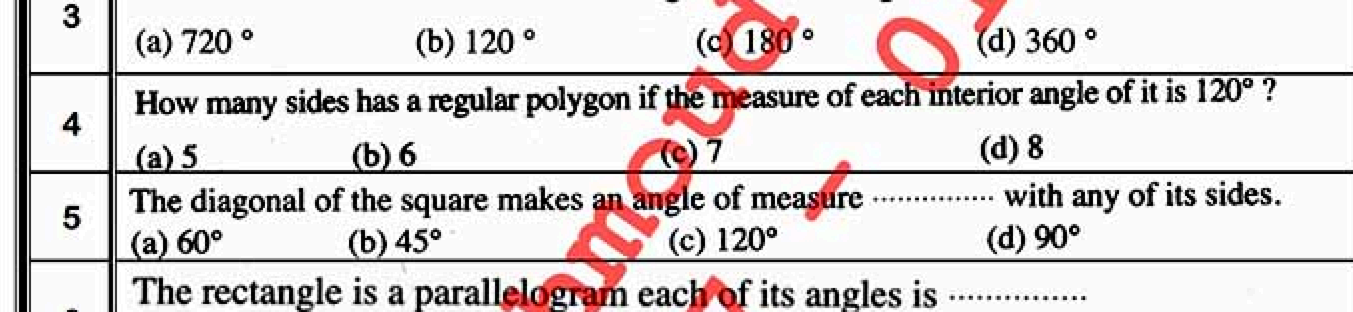


3 The sum of measures of the exterior angles of the hexagon = ...

(a) 720 ° (b) 120 ° (c) 180 ° (d) 360 °

4 How many sides has a regular polygon if the measure of each interior angle of it is 120° ?

(a) 5 (b) 6 (c) 7 (d) 8



5 The diagonal of the square makes an angle of measure .... with any of its sides.

(a) 60° (b) 45° (c) 120° (d) 90°

6 The rectangle is a parallelogram each of its angles is ....

(a) obtuse. (b) acute. (c) right. (d) straight.

7 If ABCD is a rhombus , then $\overline{AC} \perp$ ....

(a) $\overline{BD}$ (b) $\overline{AB}$ (c) $\overline{BC}$ (d) $\overline{CD}$

8 The number of axis of symmetry of a square equal ....

(a) 0 (b) 1 (c) 2 (d) 4

9 The angle with measure 70° complement angle with measure ....°

(a) 70 (b) 110 (c) 290 (d) 20

10 The area of the shaded part = ... the total area of the shape.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

11 The sum of the measures of the interior angles of a pentagon is ....

(a) 360° (b) 450° (c) 720° (d) 540°

12 The rhombus of diagonals equal in length is ....

(a) a square. (b) a rectangle. (c) a trapezium. (d) a parallelogram.

No.	Question
13	The measure of the interior angle of a regular polygon of 18 sides equals .... (a) 130° (b) 140° (c) 150° (d) 160°
14	The diagonals are equal in length and perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
15	The measure of the right angle = ... ° (a) 180 (b) 90 (c) 120 (d) 0
16	If ABCD is a parallelogram in which BC = 8 cm. and CD = 6 cm. , then its perimeter = ... (a) 14 cm. (b) 28 cm. (c) 48 cm. (d) 56 cm.
17	The side length of the rhombus which its perimeter 36 cm. is .... cm. (a) 6 (b) 9 (c) 18 (d) 4
18	If ABCD is a square , then $(AC)^2$ = ... (a) AB (b) $(AB)^2$ (c) $2\ (AB)^2$ (d) $4\ (AB)^2$
19	If Δ ABC $\equiv$ Δ XYZ , then AB = ... (a) XY (b) YZ (c) XZ (d) BC
20	The edge length of a cube whose total area is 600 cm^2. is .... cm. (a) 10 (b) 100 (c) 300 (d) 90
21	The sum of the measures of the interior angles of a triangle = ...° (a) 90 (b) 360 (c) 180 (d) 540
22	The measure of each angle of the regular hexagon is .... (a) 90° (b) 180° (c) 120° (d) 144°
23	In the .... the two diagonals are perpendicular and not equal in length. (a) square (b) rhombus (c) rectangle (d) parallelogram
24	ABCD is a parallelogram , m ($\angle$ A) = 70° , then m ($\angle$ C) = ... ° (a) 110 (b) 35 (c) 70 (d) 140
25	In a parallelogram if the adjacent sides are equal in the length , then the shape is .... (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
26	ABCD is a square , then m ($\angle$ BAC) = ...° (a) 90 (b) 60 (c) 45 (d) 30
27	If the side length of a square is 10.5 cm. , then the perimeter of this square = ... cm. (a) 40 (b) 42 (c) 50 (d) 100
28	* The parallelogram whose two diagonals are equal in length and perpendicular is called .... (a) rectangle. (b) square. (c) rhombus. (d) trapezium.

[B] : Complete the Following : -

1	The angle of measure 180° its type is ....
2	If two straight lines intersect , then the measures of each two vertically opposite angles are ....
3	A circle its radius length 10 cm. , then its circumference = ... (Consider $\pi = 3.14$)
4	Each two opposite angles in a parallelogram are ....
5	If ABCD is a parallelogram in which : m (∠ A) = 120° , then m (∠ B) = ...°
6	The rectangle is a parallelogram in which one of it's angles is ....
7	The number of axis of symmetry of square is ....
8	The two vertically opposite angles are ....
9	The opposite figure represents 3 squares each of side length 1 cm. , the perimeter of the figure = ...
10	If two opposite sides in the quadrilateral are parallel , then it is called ....
11	ABCD is parallelogram in which m (∠ A) = 100° , then m (∠ D) = ...°
12	If ABCD is rectangle and if AB = 4 cm. , BD = 5 cm. , then the area of the rectangle = ...
13	Square is a rectangle in which ....
14	The sum of the measures of the accumulative angles at a point is ....°
15	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ....
16	The measure of each interior angle of the regular pentagon = ...

17	If ABCD is a parallelogram in which m (∠ A) = 80° , then m (∠ B) = ……………
18	Two diagonals are equal in length and not perpendicular in ……………
19	The rhombus with a right angle is ……………
20	The measure of the right angle = ……………°
21	If two straight lines intersect , then the sum of measures of any two adjacent angles is ……………
22	The measure of each interior angle of the regular hexagon is ……………°
23	ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ……………
24	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ……………°
25	The length of the side of a rhombus whose perimeter is 24 cm. equals …………… cm.
26	**In the oppoiste figure :** y = ……………°
27	The measure of the straight angle equals ……………°
28	Each two opposite angles in a parallelogram are ……………
29	The sum of the measures of the angles of the quadrilateral equals ……………
30	ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ……………
31	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{3}$ m (∠ Y) , then m (∠ L) = ……………°
32	The number of axes of symmetry of the rhombus is …………… axes.
33	**In the opposite figure :** x + y = ……………°

[C] : Essay Problems : -

1

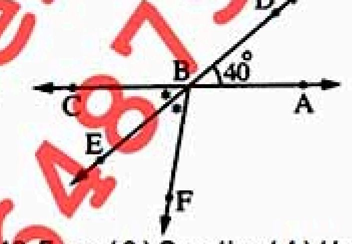
Using the geometric tools , draw the angle ABC of measure 140 , then bisect it. (don't remove arcs).

2017 Exam (12) Question (4) (b)

2

In the opposite figure :

Find : m (∠ ABF)



2016 Exam (3) Question (4) (b)

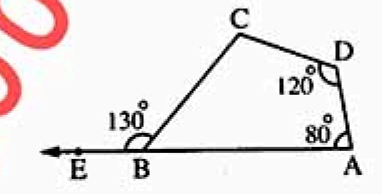
3

In the opposite figure :

m (∠ A) = 80° , m (∠ D) = 120° ,

m (∠ CBE) = 130° and B ∈ $\overrightarrow{AE}$

Find with proof : m (∠ C)



2016 Exam (6) Question (3) (a)

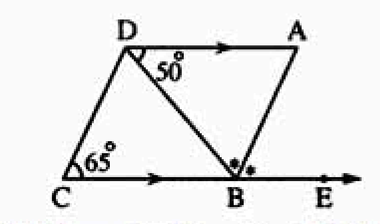
4

In the opposite figure :

$\overrightarrow{DA}$ // $\overleftrightarrow{BE}$, $\overrightarrow{BA}$ bisects ∠ DBE ,

m (∠ ADB) = 50° and m (∠ C) = 65°

Prove that : ABCD is a parallelogram.

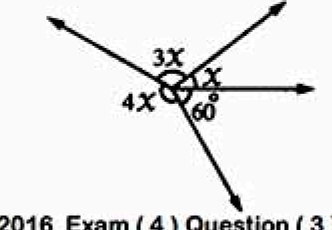


2016 Exam (13) Question (4) (a)

5

In the opposite figure :

Find : the value of x



2016 Exam (4) Question (3) (b)

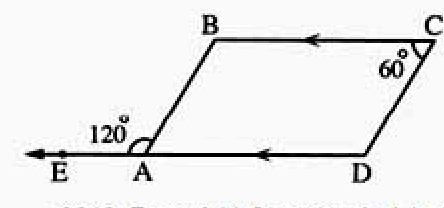
6

In the opposite figure :

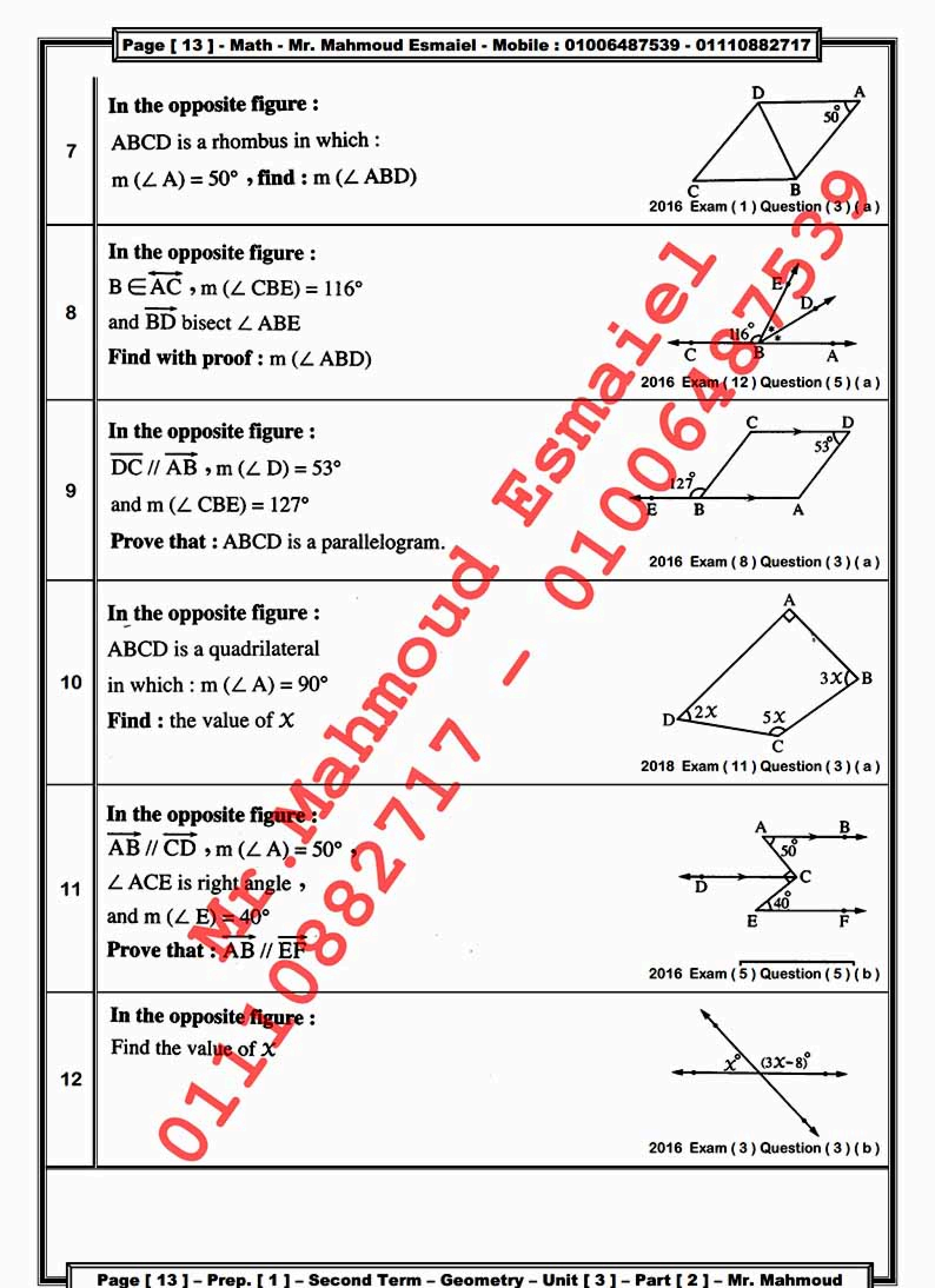
E ∈ $\overline{DA}$, m (∠ EAB) = 120°

, m (∠ C) = 60° , $\overrightarrow{DA}$ // $\overline{CB}$

Prove that : ABCD is a parallelogram



2018 Exam (4) Question (5) (a)



7

In the opposite figure :

ABCD is a rhombus in which :

m (∠ A) = 50° , **find :** m (∠ ABD)

2016 Exam (1) Question (3) (a)

8

In the opposite figure :

$B \in \overleftrightarrow{AC}$, m (∠ CBE) = 116°

and $\overrightarrow{BD}$ bisect ∠ ABE

Find with proof : m (∠ ABD)

2016 Exam (12) Question (5) (a)

9

In the opposite figure :

$\overrightarrow{DC} \parallel \overrightarrow{AB}$, m (∠ D) = 53°

and m (∠ CBE) = 127°

Prove that : ABCD is a parallelogram.

2016 Exam (8) Question (3) (a)

10

In the opposite figure :

ABCD is a quadrilateral

in which : m (∠ A) = 90°

Find : the value of x

2018 Exam (11) Question (3) (a)

11

In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m (∠ A) = 50° ,

∠ ACE is right angle ,

and m (∠ E) = 40°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$

2016 Exam (5) Question (5) (b)

12

In the opposite figure :

Find the value of x

2016 Exam (3) Question (3) (b)

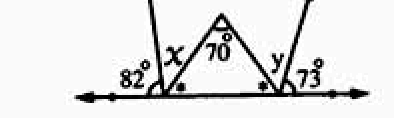
Homework

[A] : Choose The Correct Answer : -

No.	Question
1	The side length of the rhombus which its perimeter 36 cm. is .... cm. (a) 6 (b) 9 (c) 18 (d) 4
2	ABCD is a parallelogram , m (∠ A) = 70° , then m (∠ C) = ...° (a) 110 (b) 35 (c) 70 (d) 140
3	The two diagonals are equal in length and not perpendicular in .... (a) a rectangle (b) a square (c) a rhombus (d) a parallelogram
4	If the number of sides of a regular polygon is 5 and if the measure of each interior angle is (x°) , then x = ... (a) 90° (b) 108° (c) 120° (d) 180°
5	If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is .... (a) 6 (b) 4 (c) 7 (d) 8
6	In the .... the two diagonals are perpendicular and not equal in length. (a) square (b) rhombus (c) rectangle (d) parallelogram
7	The measure of the interior angle of a regular pentagon = ... (a) 900° (b) 180° (c) 540° (d) 108°
8	The pentagon has .... sides. (a) 3 (b) 4 (c) 5 (d) 6
9	The measure of the exterior angle of the equilateral triangle = ... (a) 60° (b) 90° (c) 30° (d) 120°
10	The measure of each angle of the regular hexagon is .... (a) 90° (b) 180° (c) 120° (d) 144°
11	The sum of the measures of the exterior angles of a polygon of n sides is .... (a) (n – 2) (b) (n – 2) × 180° (c) 360° (d) $\frac{(n-2) \times 180°}{n}$
12	The two bisectors of two adjacent supplementary angles included an angle of measure ....° (a) 180 (b) 45 (c) 90 (d) 0
13	The hexagon has .... sides. (a) 5 (b) 6 (c) 7 (d) 8
14	The sum of the measures of the interior angles of a triangle = ...° (a) 90 (b) 360 (c) 180 (d) 540

15	The perpendicular to one of two parallel lines is .... to the other. (a) parallel (b) equal (c) congruent (d) perpendicular	
16	The angle whose measure 90° is .... angle. (a) acute (b) right (c) obtuse (d) straight	
17	The area of the circle = ... (a) π r (b) πr^2 (c) 2 π r (d) 2 πr^2	
18	The edge length of a cube whose total area is 600 cm^2. is .... cm. (a) 10 (b) 100 (c) 300 (d) 90	
19	**The sum of the measures of the accumulative angles at a point = ...** **(a) 90° (b) 180° (c) 270° (d) 360°**	
20	The area of square of side length 3 cm is .... cm^2. (a) 9 (b) 6 (c) 12 (d) 3	
21	The perimeter of a square with side length 6 cm. = ... cm. (a) 30 (b) 36 (c) 24 (d) 216	
22	The diagonal of square divided its vertex angle in two angles of the measure of each of them is .... (a) 30° (b) 45° (c) 60° (d) 90°	
23	If Δ ABC ≡ Δ XYZ , then AB = ... (a) XY (b) YZ (c) XZ (d) BC	
24	If the side length of a square is 10.5 cm. , then the perimeter of this square = ... cm. (a) 40 (b) 42 (c) 50 (d) 100	
25	✱ The parallelogram whose two diagonals are equal in length and perpendicular is called .... (a) rectangle. (b) square. (c) rhombus. (d) trapezium.	
26	The rectangle of perpendicular diagonals is .... (a) a parallelogram. (b) a square. (c) a rhombus. (d) a trapezium.	
27	If ABCD is a rhombus and m (∠ ACB) = 32° , then m (∠ B) = ... (a) 148° (b) 161° (c) 116° (d) 32°	
28	If ABCD is a square , then $(AC)^2$ = ... (a) AB (b) $(AB)^2$ (c) 2 $(AB)^2$ (d) 4 $(AB)^2$	
29	ABCD is a square , then m (∠ BAC) = ...° (a) 90 (b) 60 (c) 45 (d) 30	
30	If ABCD is a parallelogram , then m (∠ A) = m (∠ ....) (a) B (b) C (c) D (d) nothing	

[B] : Complete the Following : -

No.	Question
1	The angle of measure 180° its type is ....
2	The two vertically opposite angles are ....
3	The measure of each interior angle of the regular pentagon = ...
4	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ...°
5	The number of axes of symmetry of the rhombus is .... axes.
6	The number of axis of symmetry of square is ....
7	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ....
8	ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...
9	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{3}$ m (∠ Y) , then m (∠ L) = ...°
10	**In the opposite figure :**  $x + y =$ ....° 
11	The rectangle is a parallelogram in which one of it's angles is ....
12	The sum of the measures of the accumulative angles at a point is ....°
13	The measure of each interior angle of the regular hexagon is ....°
14	ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ...
15	The two diagonals of the rhombus are ....
16	If ABCD is a parallelogram in which : m (∠ A) = 120° , then m (∠ B) = ...°
17	Square is a rectangle in which ....

18	If two straight lines intersect , then the sum of measures of any two adjacent angles is ....
19	The sum of the measures of the angles of the quadrilateral equals ....
20	ABCD is a parallelogram in which m (∠ A) = 130° , then m (∠ B) = ...°
21	Each two opposite angles in a parallelogram are ....
22	If ABCD is rectangle and if AB = 4 cm. , BD = 5 cm. , then the area of the rectangle = ...
23	The measure of the right angle = ...°
24	Each two opposite angles in a parallelogram are ....
25	The parallelogram whose diagonals are equal in length and not perpendicular is ....
26	A circle its radius length 10 cm. , then its circumference = ... (Consider $\pi = 3.14$)
27	ABCD is parallelogram in which m (∠ A) = 100° , then m (∠ D) = ...°
28	The rhombus with a right angle is ....
29	The measure of the straight angle equals ....°
30	The sum of the measures of the exterior angles of the convex polygon = ...
31	If two straight lines intersect , then the measures of each two vertically opposite angles are ....
32	If two opposite sides in the quadrilateral are parallel , then it is called ....
33	Two diagonals are equal in length and not perpendicular in ....
34	**In the oppoiste figure :** y = ...° (figure: y, 45°)

[C] : Essay Problems : -

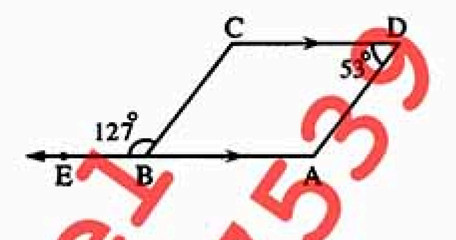
1

In the opposite figure :

$\overline{DC} \parallel \overrightarrow{AB}$, m (∠ D) = 53°

and m (∠ CBE) = 127°

Prove that : ABCD is a parallelogram.



2016 Exam (8) Question (3) (a)

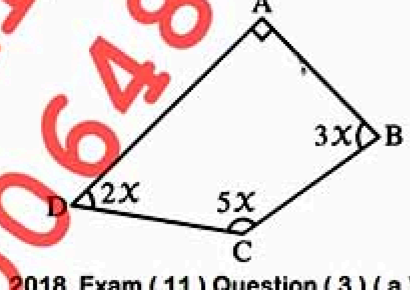
2

In the opposite figure :

ABCD is a quadrilateral

in which : m (∠ A) = 90°

Find : the value of x



2018 Exam (11) Question (3) (a)

3

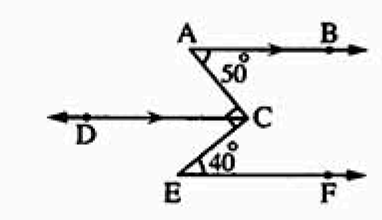
In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m (∠ A) = 50° ,

∠ ACE is right angle ,

and m (∠ E) = 40°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$

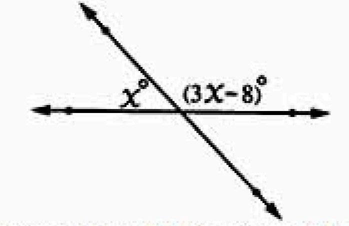


2016 Exam (5) Question (5) (b)

4

In the opposite figure :

Find the value of x



2016 Exam (3) Question (3) (b)

5

Find the number of sides of the regular polygon if the measure of its interior angle is 135°

2016 Exam (14) Question (5) (a)

6

Mention two cases of congruency of two triangles.

2017 Exam (12) Question (5) (a)

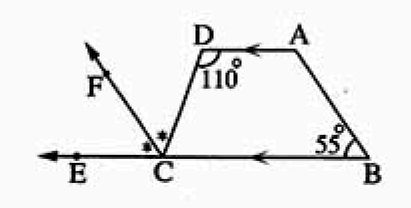
7

In the opposite figure :

$\overline{AD} \parallel \overrightarrow{BC}$, $\overrightarrow{CF}$ bisects ∠ DCE

, m (∠ ABC) = 55° , m (∠ ADC) = 110°

Prove that : $\overline{AB} \parallel \overrightarrow{CF}$



2017 Exam (11) Question (5) (b)

8

In the opposite figure :

ABCD is a square , $E \in \overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.

Model 2018 Exam (1) Question (5) (b)

9

In the opposite figure :

$F \in \overrightarrow{YZ}$, m ($\angle$ L) = 70°

, m ($\angle$ Y) = 90° and m ($\angle$ LZF) = 120°

Find : m ($\angle$ X)

2018 Exam (13) Question (3) (b)

10

In the opposite figure :

ABCD is a square

, find in degrees the value

of each of x and y

2018 Exam (12) Question (4) (a)

11

In the opposite figure :

ABCD is a parallelogram in which :

m ($\angle$ B) = 115° , AB = 8 cm.

and AD = 5 cm.

Find with proof :

(1) m ($\angle$ D)

(2) The perimeter of parallelogram ABCD

2016 Exam (14) Question (4) (a)

12

In the opposite figure :

$E \in \overline{BC}$, m ($\angle$ BAE) = 45° ,

m ($\angle$ AEB) = 70° , m ($\angle$ D) = 65°

and m ($\angle$ C) = 115°

Prove that : ABCD is a parallelogram.

2018 Exam (14) Question (4) (b)

Prep [1]

Geometry - Second Term

Unit [3] - Part [3]



Mr. Mahmoud Esmaiel

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الاســم ....

Lesson [3] : The Triangle

Theorem (1)

The sum of the measures of the interior angles of a triangle is 180°

Given ABC is a triangle

R.T.P. $m(\angle A) + m(\angle B) + m(\angle ACB) = 180°$

Construction Draw $\overleftrightarrow{CX} \parallel \overline{AB}$

Proof

$\because \angle XCY$ is a straight angle

$\therefore m(\angle XCA) + m(\angle ACB) + m(\angle BCY) = 180°$

$\because \overleftrightarrow{XY} \parallel \overline{AB}$

$\therefore m(\angle XCA) = m(\angle A)$ (alternate angles)

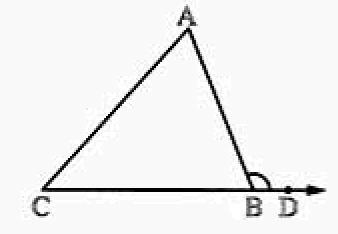
$, m(\angle YCB) = m(\angle B)$ (alternate angles)

$\therefore m(\angle A) + m(\angle ACB) + m(\angle B) = 180°$ (Q.E.D.)

The exterior angle of the triangle

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$, then $\angle ABD$ is called an exterior angle of ΔABC



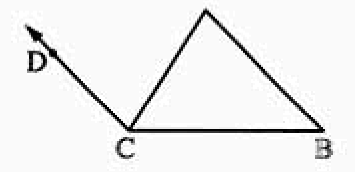
$\therefore m(\angle ABD) = m(\angle A) + m(\angle C)$

Notice That : -

In the opposite figure :

$\angle ACD$ is not an exterior angle of ΔABC

because $D \notin \overrightarrow{BC}$



The measure of the exterior angle of a triangle :

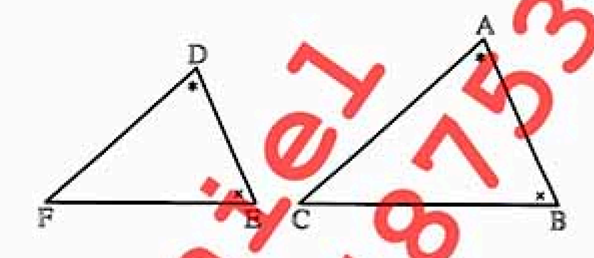
The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

Remark [1]

If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

In ΔΔ ABC and DEF ,
if m (∠ A) = m (∠ D) and m (∠ B) = m (∠ E) ,
then m (∠ C) = m (∠ F)
"You can check the truth of the previous by measuring"



Remark [2]

- If the sum of measures of two angles in a triangle equals 90° , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than 90° , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than 90° , then the third angle is acute.

Remark [3]

If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.

Theorem (2)

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

Given D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} \parallel \overline{BC}$

R.T.P. E is the midpoint of $\overline{AC}$

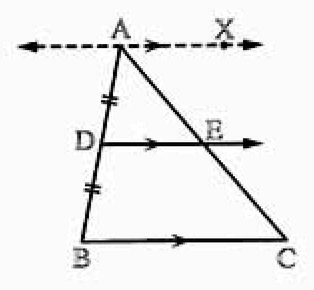
Construction Draw $\overleftrightarrow{AX} \parallel \overline{BC}$

Proof $\because \overleftrightarrow{AX} \parallel \overleftrightarrow{DE} \parallel \overline{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them at D and E respectively.

, $\because AD = DB$ $\therefore AE = EC$

$\therefore$ E is the midpoint of $\overline{AC}$ (Q.E.D.)



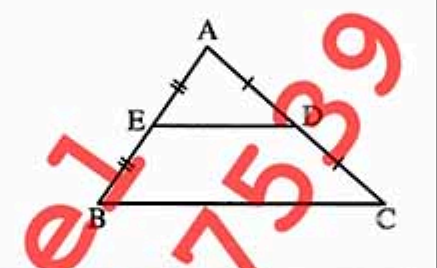
Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

In the opposite figure :

If ABC is a triangle in which D is the midpoint of $\overline{AC}$,

E is the midpoint of $\overline{AB}$, then : $\overline{ED} \parallel \overline{BC}$



Theorem (3)

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

Given ABC is a triangle , D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

R.T.P. $DH = \frac{1}{2} BC$

Construction Draw $\overrightarrow{HO} \parallel \overline{AB}$ to cut $\overline{BC}$ at O



Proof

$\because$ D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

$\therefore \overline{DH} \parallel \overline{BC}$ (corollary)

, $\because \overline{HO} \parallel \overline{AB}$ (construction) , H is the midpoint of $\overline{AC}$

$\therefore$ O is the midpoint of $\overline{BC}$ $\therefore BO = \frac{1}{2} BC$

, $\because$ The figure DHOB is a parallelogram.

$\therefore DH = BO = \frac{1}{2} BC$ (Q.E.D.)

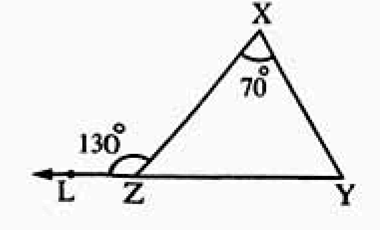
Examples :

1 **In the opposite figure :**

XYZ is a triangle , $L \in \overrightarrow{YZ}$, m (∠ XZL) = 130° ,

m (∠ X) = 70°

Find with proof : ① m (∠ Y) ② m (∠ YZX)

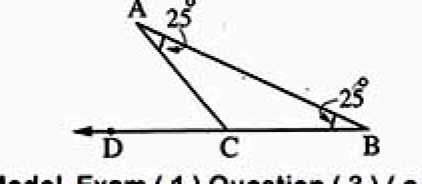


2015 Exam (9) Question (5) (a)

2 **In the opposite figure :**

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)



Model Exam (1) Question (3) (a)

3	**In the oposite figure :** $\overrightarrow{DE} \parallel \overline{BC}$, m (∠ DAB) = 82° and m (∠ D) = 32° **Find by proof :** m (∠ B) 2016 Exam (7) Question (3) (a)
4	**In the opposite figure :** $\overrightarrow{DE} \parallel \overrightarrow{OH} \parallel \overline{BC}$, m (∠ ADE) = 120° , m (∠ AOH) = 135° **Find the measures of the angles of :** Δ ABC 2014 Exam (2) Question (3) (a)
5	**In the opposite figure :** EDCF is a quadrilateral , Δ ABC is an equilateral triangle where $\overline{DB} \cap \overline{AF} = \{C\}$ **Find with proof :** m (∠ F) 2015 Exam (5) Question (4) (b)
6	**In the opposite figure :** $\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$, m (∠ A) = 30° , m (∠ EOC) = 120° , **Find :** m (∠ E) Model Exam (2) Question (4) (b)
7	**In the opposite figure :** ABC is triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , BC = 16 cm. , AC = 12 cm. **Find the perimeter of the quadrilateral :** DECF with proof 2015 Exam (15) Question (4) (a)

8	**In the opposite figure :** D , O and E are midpoints of $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ respectively. , XY = 8 cm. , YZ = 12 cm. , XZ = 10 cm. **Find with proof :** The perimeter of Δ EOD 2017 Exam (9) Question (5) (a)
9	**In the opposite figure :** E , D and F are the midpoints of $\overline{AC}$, $\overline{AB}$ and $\overline{BC}$ respectively. ED = 6 cm. , DF = 3 cm. and EF = 5 cm. **Find with proof :** The perimeter of Δ ABC 2017 Exam (5) Question (3) (b)
10	**In the opposite figure :** $\overrightarrow{AZ}$ // $\overline{YD}$ // $\overline{XE}$ // $\overline{CB}$, AY = YX = XC , AB = 18 cm. , **Find :** The length of $\overline{EB}$ Model Exam (1) Question (4) (b)
11	**In the opposite figure :** D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, DX = XF , EY = FY , BC = 12 cm. **Find :** The length of $\overline{XY}$ 2017 Exam (5) Question (4) (b)

Solutions

1	$\because$ $\angle$ XZL is an exterior angle of Δ XYZ $\therefore$ m ($\angle$ Y) = 130° – 70° = 60° (First req.) $\therefore$ From Δ XYZ : m ($\angle$ YZX) = 180° – (70° + 60°) = 50° (Second req.)

2	$\because$ m ($\angle$ A) = m ($\angle$ B) = 25° , $\angle$ ACD is an exterior angle of Δ ABC $\therefore$ m ($\angle$ ACD) = 25° + 25° = 50° (The req.)

No.	Answer
3	$\because \overleftrightarrow{DE} \parallel \overline{BC}$, $\overleftrightarrow{CD}$ is a transversal $\therefore m(\angle C) = m(\angle D) = 32^\circ$ (alternate angles) , $\because \angle DAB$ is an exterior angle of ΔABC $\therefore m(\angle B) = 82^\circ - 32^\circ = 50^\circ$ (The req.)
4	$\because \overrightarrow{OH} \parallel \overline{BC}$, $\overleftrightarrow{OB}$ is a transversal $\therefore m(\angle B) + m(\angle O) = 180^\circ$ (Two interior angles in the same side of the transversal) $\therefore m(\angle B) = 180^\circ - 135^\circ = 45^\circ$ $\because \overrightarrow{DE} \parallel \overline{BC}$, $\overleftrightarrow{CD}$ is a transversal $\therefore m(\angle C) + m(\angle D) = 180^\circ$ (Two interior angles in the same side of the transversal) $\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$ In ΔABC : $\therefore m(\angle BAC) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$ (The req.)
5	$\because \Delta ABC$ is an equilateral triangle. $\therefore m(\angle ACB) = \frac{180^\circ}{3} = 60^\circ$ $\therefore m(\angle DCF) = m(\angle ACB) = 60^\circ$ (V.O.A) From quadrilateral EDCF : $\therefore m(\angle F) = 360^\circ - (60^\circ + 120^\circ + 118^\circ)$ $= 62^\circ$ (The req.)
6	In ΔABC : $m(\angle ACB) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$, $\because \overline{BD} \cap \overline{AO} = \{C\}$ $\therefore m(\angle ACB) = m(\angle OCD) = 60^\circ$ (V.O.A) $\therefore m(\angle E) = 360^\circ - (60^\circ + 120^\circ + 90^\circ) = 90^\circ$ (The req.)
7	$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$ $\therefore DE = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6$ cm. , $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$ $\therefore DF = \frac{1}{2} BC = \frac{1}{2} \times 16 = 8$ cm. , $\because$ E is the midpoint of $\overline{BC}$ $\therefore CE = \frac{1}{2} BC = \frac{1}{2} \times 16 = 8$ cm. , $\because$ F is the midpoint of $\overline{AC}$ $\therefore CF = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6$ cm. $\therefore$ The perimeter of DECF $= 6 + 8 + 6 + 8$ $= 28$ cm. (The req.)
8	In ΔXYZ : $\because$ E is the midpoint of $\overline{XZ}$, O is the midpoint of $\overline{ZY}$ $\therefore EO = \frac{1}{2} XY = \frac{1}{2} \times 8 = 4$ cm. , $\because$ O is the midpoint of $\overline{ZY}$, D is the midpoint of $\overline{XY}$ $\therefore OD = \frac{1}{2} \times ZX = \frac{1}{2} \times 10 = 5$ cm. , $\because$ D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$ $\therefore ED = \frac{1}{2} YZ = \frac{1}{2} \times 12 = 6$ cm. $\therefore$ The perimeter of $\Delta EOD = 4 + 5 + 6$ $= 15$ cm. (The req.)
9	$\because$ In ΔABC : $\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ $\therefore BC = 2\,DE = 2 \times 6 = 12$ cm. , $\because$ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$ $\therefore AC = 2\,DF = 2 \times 3 = 6$ cm. , $\because$ E is the midpoint of $\overline{AC}$, F is the midpoint of $\overline{BC}$ $\therefore AB = 2\,EF = 2 \times 5 = 10$ cm. $\therefore$ the perimeter of $\Delta ABC = 12 + 6 + 10$ $= 28$ cm. (The req.)
10	$\because \overleftrightarrow{AZ} \parallel \overleftrightarrow{YD} \parallel \overleftrightarrow{XE} \parallel \overleftrightarrow{CB}$, $AY = YX = XC$ $\therefore AD = DE = EB$ $\therefore EB = \frac{18}{3} = 6$ cm. (The req.)
11	In ΔABC : $\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ $\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 12 = 6$ cm. , In ΔEFD : $\because$ X is the midpoint of $\overline{FD}$, Y is the midpoint of $\overline{EF}$ $\therefore XY = \frac{1}{2} ED = \frac{1}{2} \times 6 = 3$ cm. (The req.)

Exercises

[A] : Choose The Correct Answer : -

1 The sum of measures of the angles of a triangle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

2 The parallelogram whose two diagonals are .... is called a rectangle.

(a) parallel (b) perpendicular (c) equal in length (d) bisect each other

3 The rectangle is a parallelogram each of its angles is ....

(a) obtuse. (b) acute. (c) right. (d) straight.

4 The two diagonals are equal in length and not perpendicular in ....

(a) a rectangle (b) a square (c) a rhombus (d) a parallelogram

5 The sum of measures of the exterior angles of the hexagon = ...

(a) 720 ° (b) 120 ° (c) 180 ° (d) 360 °

6 The area of the circle = ...

(a) πr (b) πr^2 (c) $2\pi r$ (d) $2\pi r^2$

7 **In the opposite figure :**

X and Y are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, XY = 10 cm. , then BC = ... cm.

(a) 5 (b) 10

(c) 20 (d) 30

A
X
Y
10 cm.
B
C

8 The length of the line segment joining the midpoints of two sides of a triangle is equal to .... the length of the third side.

(a) half (b) quarter (c) twice (d) third

9 Any triangle has at least two .... angles.

(a) reflex (b) obtuse (c) acute (d) right

10 The diagonals are equal in length and perpendicular in ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

11 In the .... the two diagonals are perpendicular and not equal in length.

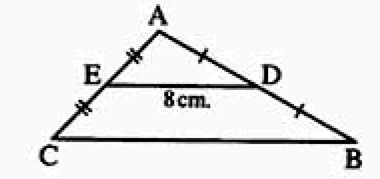
(a) square (b) rhombus (c) rectangle (d) parallelogram

12 The measure of the exterior angle of the equilateral triangle = ...

(a) 60° (b) 90° (c) 30° (d) 120°

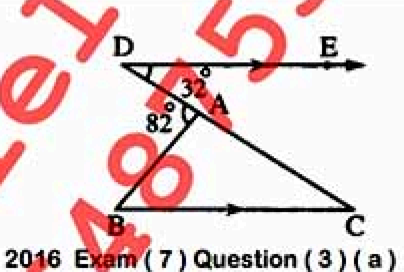
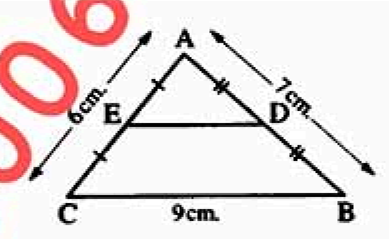
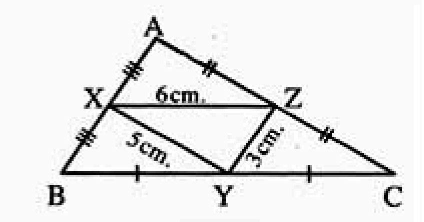
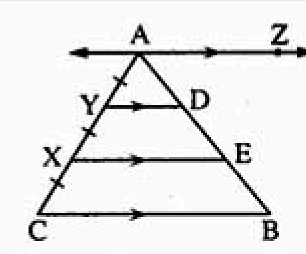
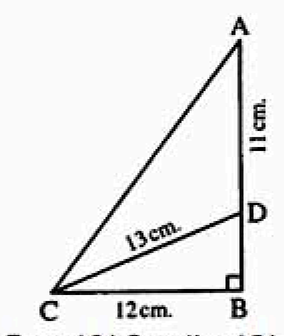
No.	Question
13	The two bisectors of two adjacent supplementary angles included an angle of measure ....° (a) 180 (b) 45 (c) 90 (d) 0
14	**In the opposite figure :** X , Y are midpoints of $\overline{AB}$, $\overline{AC}$ respectively , BC = 10 cm. , then XY = ... cm. (a) 5 (b) 20 (c) 10 (d) 30
15	The length of the line segment joining between two midpoints of two sides of a triangle = ... length of its third side. (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
16	The sum of the interior angles of an isosceles triangle = ... (a) 180° (b) 90° (c) 60° (d) 45°
17	If ABCD is a square , then $(AC)^2$ = ... (a) AB (b) $(AB)^2$ (c) $2\ (AB)^2$ (d) $4\ (AB)^2$
18	ABCD is a square , then m (∠ BAC) = ...° (a) 90 (b) 60 (c) 45 (d) 30
19	The measure of the interior angle of a regular pentagon = ... (a) 900° (b) 180° (c) 540° (d) 108°
20	If the number of sides of a regular polygon is 5 and if the measure of each interior angle is (x°) , then x = ... (a) 90° (b) 108° (c) 120° (d) 180°
21	The perpendicular to one of two parallel lines is .... to the other. (a) parallel (b) equal (c) congruent (d) perpendicular
22	**In the opposite figure :** ED : BC = ... (a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4
23	The line segment joining the midpoints of two sides of a triangle is .... the third side. (a) perpendicular to (b) equal to (c) parallel to (d) bisect to
24	The sum of the measures of the exterior angles of triangle = ... (a) 90° (b) 180° (c) 360° (d) 120°

[B] : Complete the Following : -

1	Any triangle has at least two .... interior angles.
2	The number of axis of symmetry of the isosceles triangle = ...
3	The sum of the measures of the interior angles of a triangle = ...°
4	The measure of the exterior angle of a triangle is ....
5	The measure of the exterior angle of any vertex of the equilateral triangle = ...°
6	The measure of the exterior angle of a triangle is equal to the sum of ....
7	The ray drawn from the midpoint of a side of a triangle parallel to another side .... the third side.
8	The ray drawn parallel to one side of triangle and passing through the midpoint of another side ....
9	The line segment joining midpoints of two sides of a triangle is ....
10	The line segment joining between two midpoints of two sides of triangle is parallel to ....
11	The line segment joining the midpoint of two sides of a triangle is .... the third side.
12	The length of the line segment joining the midpoints of two sides of a triangle is equal to .... the third side.
13	The length of the line segment joining the midpoints of two sides of a triangle equals ....
14	**In the opposite figure :** If ED = 8 cm. , then BC = ... cm. 

15	**In the opposite figure :** If BC = 22 cm. , then ED = …………… cm. A, E, D, C, B, 22cm.
16	**In the opposite figure :** $\overline{XY}$ // …………… A, X, Y, C, B
17	**In the opposite figure :** ABC is an equilateral triangle where D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively , and $\overline{AE} \cap \overline{BF} \cap \overline{CD} = \{M\}$ Then the image of Δ AMD by reflection in $\overleftrightarrow{AE}$ is …………… A, F, D, M, C, E, B
18	The angle of measure 180° its type is ……………
19	The measure of the straight angle equals ……………°
20	The measure of the right angle = ……………°
21	The sum of the measures of the accumulative angles at a point is ……………°
22	The two vertically opposite angles are ……………
23	If two straight lines intersect , then the measures of each two vertically opposite angles are ……………
24	Every two vertically opposite angles are …………… in measure.
25	Each two opposite angles in a parallelogram are ……………

[C] : Essay Problems : -

1	**Prove that :** the ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side. 2018 Exam (4) Question (3) (a)
2	**In the oposite figure :** $\overrightarrow{DE}$ // $\overline{BC}$, m (∠ DAB) = 82° and m (∠ D) = 32° **Find by proof :** m (∠ B)  2016 Exam (7) Question (3) (a)
3	**In the opposite figure :** ABC is a triangle in which D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , AB = 7 cm. , BC = 9 cm. and AC = 6 cm. **Find :** the perimeter of Δ ADE  2016 Exam (11) Question (3) (b)
4	**In the opposite figure :** X , Y , Z are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively. If XY = 5 cm. , YZ = 3 cm. and XZ = 6 cm. Find with proof the perimeter of Δ ABC  2018 Exam (6) Question (3) (b)
5	**In the opposite figure :** $\overleftrightarrow{AZ}$ // $\overline{YD}$ // $\overline{XE}$ // $\overline{BC}$, AY = YX = XC and AB = 12 cm. **Find :** the length of $\overline{AD}$  2016 Exam (9) Question (5) (a)
6	**In the opposite figure :** ABC is a triangle in which m (∠ B) = 90° , D ∈ $\overline{AB}$ such that AD = 11 cm. If BC = 12 cm. , DC = 13 cm. , **find :** the length of each of $\overline{BD}$ and $\overline{AC}$  2016 Exam (3) Question (5) (a)

7	**In the opposite figure :** $\overrightarrow{AB} // \overrightarrow{DE}$, m (∠ D) = 70° and m (∠ DOA) = 120° **Find with proof :** m (∠ A) 2016 Exam (9) Question (3) (b)
8	**In the opposite figure :** Δ ABC is an equilateral triangle whose side length 6 cm. , D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{BA}$ **Prove that :** Δ EBD is an equilateral triangle and find its perimeter. 2017 Exam (14) Question (3) (a)
9	**In the opposite figure :** The points D , E and F are midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively , BC = 5 cm. and AC = 7 cm. **Find by proof :** the perimeter of the quadrilateral DECF 2016 Exam (7) Question (3) (b)
10	**In the opposite figure :** $\overrightarrow{AZ} // \overline{YD} // \overline{XE} // \overline{CB}$, AY = YX = XC , AB = 18 cm. , **Find :** The length of $\overline{EB}$ Model 2018 Exam (1) Question (4) (b)
11	**In the opposite figure :** AE = EB , $\overline{EX} // \overline{BC}$, $\overline{XY} // \overline{CD}$ and AD = 6 cm. **Find :** The length of $\overline{AY}$ 2017 Exam (4) Question (4) (a)
12	**In the opposite figure :** D , E are midpoints of $\overline{AB}$, $\overline{AC}$ respectively AD = 3 cm. , AE = 2 cm. , DE = 4 cm. **Find :** the perimeter of Δ ABC

2018 Exam (5) Question (4) (a)

13

In the opposite figure :

$\overrightarrow{DE} \parallel \overline{CB}$,

m ($\angle$ D) = 60° ,

m ($\angle$ C) = 50°

Find : m ($\angle$ DAC)

2016 Exam (6) Question (4) (b)

14

In the opposite figure :

ABC is a triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

BC = 12 cm. , and AC = 10 cm.

Find : the perimeter of the quadrilateral DECF

2018 Exam (14) Question (5) (a)

15

In the opposite figure :

AB = 10 cm. , BC = 16 cm. , and AC = 14 cm. ,

D , E and F are the midpoints of

$\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively.

Prove that :

The perimeter of Δ DEF = 20 cm.

2018 Exam (1) Question (3) (b)

16

In the opposite figure :

AC = BC in the triangle ABC , E is the midpoint of $\overline{AB}$,

$\overline{EF} \parallel \overline{AC}$, H and G are the midpoints of $\overline{BD}$,

$\overline{CD}$ respectively

Prove that : EF = GH = BF

2016 Exam (1) Question (5) (b)

17

In the opposite figure :

m ($\angle$ A) = m ($\angle$ B) = 25°

Find : m ($\angle$ ACD)

Model 2018 Exam (1) Question (3) (a)

Homework

[A] : Choose The Correct Answer : -

1	The number of axis of symmetry of a square equal …………… (a) 0 (b) 1 (c) 2 (d) 4
2	The parallelogram whose diagonals are perpendicular to each other and not equal in length is called …………… (a) a square (b) a rectangle (c) a rhombus (d) a trapezium
3	The rectangle of perpendicular diagonals is …………… (a) a parallelogram. (b) a square. (c) a rhombus. (d) a trapezium.
4	The hexagon has …………… sides. (a) 5 (b) 6 (c) 7 (d) 8
5	The edge length of a cube whose total area is 600 cm^2. is …………… cm. (a) 10 (b) 100 (c) 300 (d) 90
6	The measure of the right angle = …………… ° (a) 180 (b) 90 (c) 120 (d) 0
7	In Δ ABC , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 8 cm. , then DE = …………… cm. (a) 16 (b) 8 (c) 4 (d) 2
8	The measure of the exterior angle of the equilateral triangle = …………… (a) 60° (b) 90° (c) 30° (d) 120°
9	The diagonal of square divided its vertex angle in two angles of the measure of each of them is …………… (a) 30° (b) 45° (c) 60° (d) 90°
10	In a parallelogram if the adjacent sides are equal in the length , then the shape is …………… (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
11	If ABCD is a parallelogram , then m (∠ A) = m (∠ …………) (a) B (b) C (c) D (d) nothing
12	The diagonal of the square makes an angle of measure …………… with any of its sides. (a) 60° (b) 45° (c) 120° (d) 90°
13	The sum of the measures of the accumulative angles at a point = …………… (a) 90° (b) 180° (c) 270° (d) 360°
14	The angle whose measure 90° is …………… angle. (a) acute (b) right (c) obtuse (d) straight

15 If X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ in Δ ABC and XY = 3 cm.
, then BC = ... cm.

(a) 3 (b) 5 (c) 6 (d) 9

16 Any triangle has at least two .... interior angles.

(a) right (b) obtuse (c) acute (d) reflex

17 The perimeter of a square with side length 6 cm. = ... cm.

(a) 30 (b) 36 (c) 24 (d) 216

18 In Δ ABC if : m ($\angle$ A) > m ($\angle$ B) + m ($\angle$ C) , then the angle A is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

19 In Δ ABC if : X , Y are the midpoints of $\overline{AC}$ and $\overline{BC}$ respectively , then $\overline{XY}$ // ....

(a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{CY}$

20 ✱ The triangle contains two .... angles at least

(a) acute (b) obtuse (c) right (d) reflex

21 The area of square of side length 3 cm is .... cm^2.

(a) 9 (b) 6 (c) 12 (d) 3

22 If ABCD is a rhombus , then $\overline{AC} \perp$ ....

(a) $\overline{BD}$ (b) $\overline{AB}$ (c) $\overline{BC}$ (d) $\overline{CD}$

23 ABCD is a parallelogram in which : m ($\angle$ A) = 60° , then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 120°

24 How many sides has a regular polygon if the measure of each interior angle of it is 120° ?

(a) 5 (b) 6 (c) 7 (d) 8

25 The sum of the measures of the exterior angles of a polygon of n sides is ....

(a) (n – 2) (b) (n – 2) × 180° (c) 360° (d) $\frac{(n-2) \times 180°}{n}$

26 The pentagon has .... sides.

(a) 3 (b) 4 (c) 5 (d) 6

27 The smallest number of the acute angle in any triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

28 The right-angled triangle has .... right angle.

(a) 1 (b) 2 (c) 0 (d) 3

29 **In the oppoiste figure :**

m ($\angle$ A) = m ($\angle$ C) , x = ...°

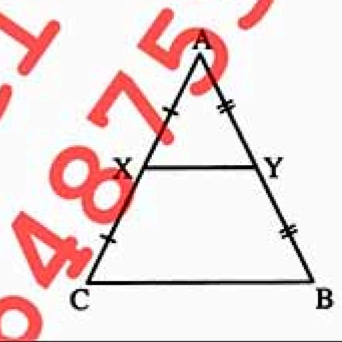
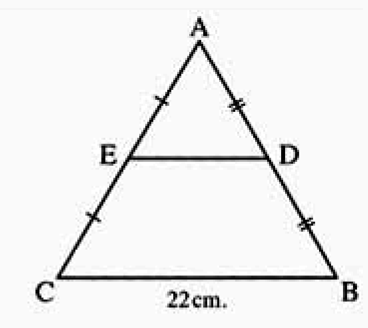
(a) 50° (b) 130°

(c) 25° (d) 180°

30 In Δ ABC, if m ($\angle$ C) : m ($\angle$ A) : m ($\angle$ B) = 1 : 2 : 4 , then $\angle$ B is ....

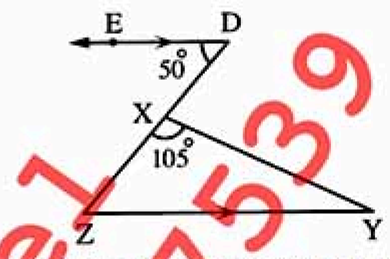
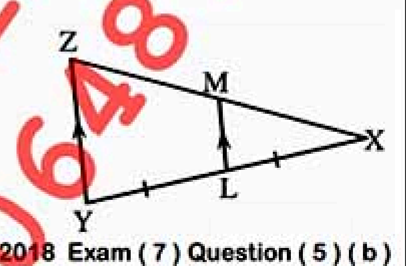
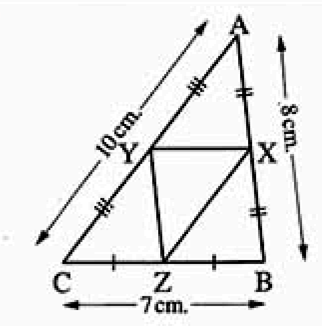
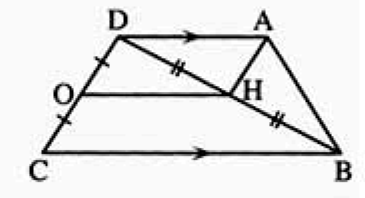
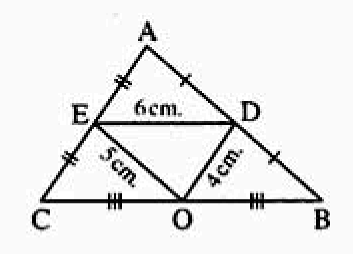
(a) an obtuse (b) an acute (c) a right (d) otherwise

[B] : Complete the Following : -

No.	Question
1	The ray drawn parallel to one side of triangle and passing through the midpoint of another side ……………
2	**In the opposite figure :**  $\overline{XY}$ // …………… 
3	Every two vertically opposite angles are …………… in measure.
4	The measure of each interior angle of the regular pentagon = ……………
5	If ABCD is a parallelogram in which : m (∠ A) = 120° , then m (∠ B) = ……………°
6	The number of axes of symmetry of the rhombus is …………… axes.
7	The ray drawn from the midpoint of a side of a triangle parallel to another side …………… the third side.
8	**In the opposite figure :** If BC = 22 cm. , then ED = …………… cm. 
9	If two straight lines intersect , then the measures of each two vertically opposite angles are ……………
10	The measure of each interior angle of the regular hexagon is ……………°
11	Any triangle has at least two …………… interior angles.
12	ABCD is parallelogram in which m (∠ A) = 100° , then m (∠ D) = ……………°

13	The two diagonals of the rhombus are ....
14	The measure of the exterior angle of a triangle is equal to the sum of ....
15	**In the opposite figure :** If ED = 8 cm. , then BC = ... cm.
16	The two vertically opposite angles are ....
17	The sum of the measures of the angles of the quadrilateral equals ....
18	If ABCD is a parallelogram in which m (∠ A) = 80° , then m (∠ B) = ...
19	The rectangle is a parallelogram in which one of it's angles is ....
20	The measure of the exterior angle of any vertex of the equilateral triangle = ...°
21	The length of the line segment joining the midpoints of two sides of a triangle equals ....
22	The sum of the measures of the accumulative angles at a point is ....°
23	The sum of the measures of the exterior angles of the convex polygon = ...
24	ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...
25	If ABCD is rectangle and if AB = 4 cm. , BD = 5 cm. , then the area of the rectangle = ...
26	The measure of the exterior angle of a triangle is ....
27	The length of the line segment joining the midpoints of two sides of a triangle is equal to .... the third side.
28	The measure of the right angle = ...°
29	A circle its radius length 10 cm. , then its circumference = ... (Consider $\pi = 3.14$)

[C] : Essay Problems : -

1	**In the opposite figure :** $\overrightarrow{DE} \parallel \overline{YZ}$, m (∠ ZDE) = 50° , m (∠ YXZ) = 105° **Find :** m (∠ Z) , m (∠ Y) , m (∠ YXD)  Model 2018 Exam (1) Question (4) (a)
2	**In the opposite figure :** L is a midpoint of $\overline{XY}$, $\overline{LM} \parallel \overline{YZ}$, XZ = 10 cm. **Find :** the length of $\overline{XM}$  2018 Exam (7) Question (5) (b)
3	**In the opposite figure :** ABC is a triangle in which X , Y and Z are midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively. AB = 8 cm. , AC = 10 cm. , BC = 7 cm. **Find by proof :** The perimeter of Δ XYZ  2017 Exam (7) Question (4) (a)
4	**In the opposite figure :** $\overline{AD} \parallel \overline{BC}$, AD = $\frac{1}{2}$ BC , H and O are midpoints of $\overline{DB}$ and $\overline{DC}$ respectively **Prove that :** AHOD is a parallelogram  2018 Exam (2) Question (4) (a)
5	**In the opposite figure :** Δ ABC , in which D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{BC}$, ED = 6 cm. , OD = 4 cm. and EO = 5 cm. **Find :** the perimeter of Δ ABC  2016 Exam (9) Question (5) (b)

6

In the opposite figure :

$\overrightarrow{DE} \parallel \overline{BC}$, m ($\angle$ D) = 100°

, m ($\angle$ C) = 30° and A $\in \overline{DB}$

Find : m ($\angle$ BAC)

2018 Exam (11) Question (4) (a)

7

In the opposite figure :

ABCD is a parallelogram its

diagonals are intersect at M

, $\overrightarrow{ME} \parallel \overline{AB}$ prove that BE = EC

2018 Exam (4) Question (3) (b)

8

In the opposite figure :

$\overrightarrow{BA} \parallel \overline{CD}$, m ($\angle$ B) = 50°

and m ($\angle$ D) = 60°

Find with proof :

m ($\angle$ CED)

2016 Exam (4) Question (3) (a)

9

In the opposite figure :

D and E are midpoints of $\overline{AB}$ and $\overline{AC}$

, AB = 12 cm. , BC = 18 cm. and AC = 16 cm.

Find with proof : the perimeter of figure ECBD

2016 Exam (6) Question (5) (b)

10

In the opposite figure :

ABC is a triangle in which D , E , O are midpoints

of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ resectively , $\overline{DE} \parallel \overline{AC}$

AB = 6 cm. , BC = 8 cm. , AC = 10 cm.

Find with prove the perimeter of : Δ EDO

2018 Exam (2) Question (3) (a)

11

Complete :

The line segment joining the midpoints of two sides of a triangle is ....

2017 Exam (1) Question (4) (a)

12

Prove that : The sum of the measures of the interior angles of a triangle is 180°

2016 Exam (11) Question (3) (a)

No.	Question
13	**In the opposite figure :** $AD = DB$, $AE = EC$, $\overline{AX} \parallel \overline{BC}$, $\overrightarrow{DE} \cap \overline{XC} = \{Y\}$, **prove that :** Y is the midpoint of $\overline{XC}$ 2018 Exam (12) Question (5) (a)
14	**In the opposite figure :** $\overrightarrow{EO} \parallel \overline{CD}$, $m(\angle E) = 50°$, $m(\angle C) = 30°$, Find the measures of angles of ΔABC , $m(\angle ABD)$ Model 2018 Exam (2) Question (5) (a)
15	**In the opposite figure :** D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, $\overline{DF} \cap \overline{BC} = \{X\}$, $DX = XF$ and $BC = 12$ cm. **Find :** the length of $\overline{XY}$ 2018 Exam (13) Question (4) (a)
16	**In the opposite figure :** $m(\angle A) = 85°$, $m(\angle D) = 100°$, $m(\angle C) = 50°$, $m(\angle E) = 35°$ **Find with proof each of :** (1) $m(\angle DOB)$ (2) $m(\angle B)$ 2017 Exam (2) Question (4) (a)
17	**In the opposite figure :** X is the midpoint of $\overline{AB}$, $Y \in \overline{CD}$, $Z \in \overline{CE}$, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$ Is $CZ = ZE$? giving reason Model 2018 Exam (2) Question (5) (b)

Prep [1]

Geometry - Second Term

Unit [3] - Part [4]



Mr. Mahmoud Esmaiel

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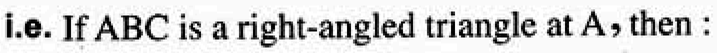
الاسم ....

Lesson [4] : Pythagoras Theorem

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



i.e. If ABC is a right-angled triangle at A, then :

$(BC)^2 = (AB)^2 + (AC)^2$



• **From the previous relation, we can deduce the following two relations :**

$(AB)^2 = (BC)^2 - (AC)^2$ $\quad$ $(AC)^2 = (BC)^2 - (AB)^2$

Remark [1]

You can get three numbers representing the lengths of sides of a right-angled triangle as follows :

1. If M is an even number bigger than 2, then the numbers M, $\left(\frac{M}{2}\right)^2 - 1$, $\left(\frac{M}{2}\right)^2 + 1$

2. If M is an odd number bigger than 2, then the numbers M, $\frac{M^2 - 1}{2}$, $\frac{M^2 + 1}{2}$ represent three lengths of sides of a right-angled triangle as shown in the following table :

For Example :

in △ ABC, AB = 3 cm, BC = 4 cm, m ∠(B) = 90, AC = $\sqrt{(AB)^2 + (BC)^2}$ = 5 cm

in △ ABC, AB = 6 cm, BC = 8 cm, m ∠(B) = 90, AC = $\sqrt{(AB)^2 + (BC)^2}$ = 10 cm

in △ ABC, AC = 15 cm, BC = 12 cm, m ∠(B) = 90, AB = $\sqrt{(AC)^2 - (BC)^2}$ = 9 cm

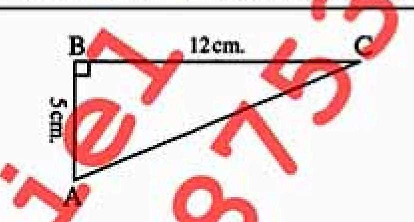
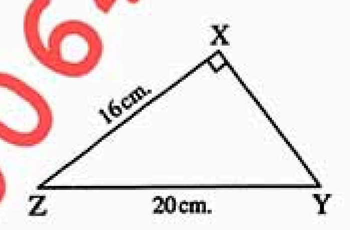
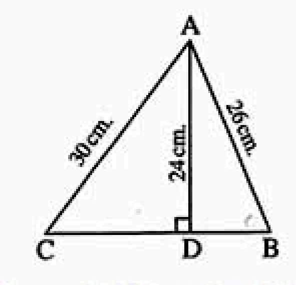
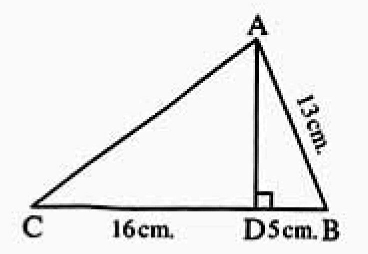
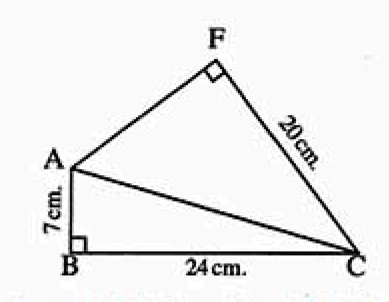
in △ ABC, AC = 25 cm, BC = 20 cm, m ∠(B) = 90, AB = $\sqrt{(AC)^2 - (BC)^2}$ = 15 cm

in △ ABC, AC = 25 cm, AB = 15 cm, m ∠(B) = 90, BC = $\sqrt{(AC)^2 - (AB)^2}$ = 20 cm

in △ ABC, AC = 10 cm, AB = 6 cm, m ∠(B) = 90, BC = $\sqrt{(AC)^2 - (AB)^2}$ = 8 cm



Examples :

1	ABCD is a rectangle in which AB = 6 cm. , AC = 10 cm. **Find :** The length of $\overline{AD}$ **2017 Exam (9) Question (3) (b)**
2	**In the opposite figure :** ABC is a triangle in which m (∠ B) = 90° , AB = 5 cm. , BC = 12 cm. **Find :** AC  **2017 Exam (1) Question (5) (a)**
3	**In the opposite figure :** XYZ is a triangle in which m (∠ X) = 90° , YZ = 20 cm. , XZ = 16 cm. **Find :** The length of $\overline{XY}$  **2017 Exam (6) Question (4) (a)**
4	**In the opposite figure :** ABC is a triangle in which : AD ⊥ BC If AD = 24 cm. , AB = 26 cm. and AC = 30 cm. **Find :** BC  **2017 Exam (4) Question (4) (b)**
5	**In the opposite figure :** $\overline{AD} \perp \overline{BC}$, BD = 5 cm. , DC = 16 cm. , AB = 13 cm. **Find the length of :** $\overline{AD}$, $\overline{AC}$  **2015 Exam (5) Question (3) (b)**
6	**In the opposite figure :** ABCF is a quadrilateral in which : m (∠ ABC) = m (∠ AFC) = 90° , AB = 7 cm. , BC = 24 cm. , FC = 20 cm. **Find :** AC and AF  **2017 Exam (11) Question (3) (b)**

No.	Question
7	**In the opposite figure :** ABC is a triangle in which m (∠ B) = 90° , D ∈ $\overline{AB}$ Such that : AD = 11 cm. , if BC = 12 cm. , DC = 13 cm. , **Find :** The length of each of $\overline{BD}$ and $\overline{AC}$ Model Exam (8) Question (5) (b)
8	**In the opposite figure :** Δ ABC in which m (∠ B) = 90° , $\overline{AE}$ // $\overline{BC}$, if BC = 8 cm. , AC = 17 cm. , D ∈ $\overline{AB}$: BD = 3 cm. , AE = 2 BC **Find :** The length of each of $\overline{AD}$ and $\overline{ED}$ 2017 Exam (8) Question (3) (a)
9	Find the length of the diagonal of a rectangle whose area 48 cm^2 and of width 6 cm. 2017 Exam (10) Question (3) (b)
10	**In the opposite figure :** ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, m (∠ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm. **Find :** The length of DC 2017 Exam (2) Question (5) (b)
11	**In the opposite figure :** ABCD is a quadrilateral in which m (∠ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm. and $\overline{AD}$ // $\overline{BC}$ **Find with proof :** The length of $\overline{DC}$ 2017 Exam (10) Question (5) (b)

Solutions

1

to get the length of the rectangle ,

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (BC)^2 = (AC)^2 - (AB)^2$

$= 100 - 36 = 64$

$\therefore BC = \sqrt{64} = 8$ cm. (The req.)

2

In Δ ABC : $\because$ ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 25 + 144 = 169$

$\therefore AC = \sqrt{169} = 13$ cm. (the req.)

3

In Δ XYZ : $\because$ m ($\angle$ X) = 90°

$\therefore (XY)^2 = (YZ)^2 - (XZ)^2 = (20)^2 - (16)^2 = 144$

$\therefore XY = \sqrt{144} = 12$ cm. (The req.)

4

In Δ ABD : $\because \overline{AD} \perp \overline{BC}$ $\therefore$ m ($\angle$ ADB) = 90°

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

In Δ ACD : $\because$ m ($\angle$ ADC) = 90°

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

$\therefore BC = BD + CD = 10 + 18 = 28$ cm. (The req.)

5

In Δ ABD : $\because$ m ($\angle$ ADB) = 90°

$\therefore (AD)^2 = (AB)^2 - (BD)^2 = (13)^2 - (5)^2 = 144$

$\therefore AD = \sqrt{144} = 12$ cm.

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

$\therefore (AC)^2 = (AD)^2 + (CD)^2 = (12)^2 + (16)^2 = 400$

$\therefore AC = \sqrt{400} = 20$ cm. (The req.)

6

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore AC = \sqrt{625} = 25$ cm.

, In Δ AFC : $\because$ m ($\angle$ F) = 90°

$\therefore (AF)^2 = (AC)^2 - (FC)^2 = (25)^2 - (20)^2 = 225$

$\therefore AF = \sqrt{225} = 15$ cm. (the req.)

7

In Δ BCD : $\because$ m ($\angle$ B) = 90°

$\therefore (BD)^2 = (DC)^2 - (BC)^2 = 169 - 144 = 25$

$\therefore BD = \sqrt{25} = 5$ cm.

$\therefore AB = 5 + 11 = 16$ cm.

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 256 + 144 = 400$

$\therefore AC = \sqrt{400} = 20$ cm. (The req.)

8

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 289 - 64 = 225$

$\therefore AB = \sqrt{225} = 15$ cm.

$\therefore AD = AB - DB = 15 - 3 = 12$ cm.

, $\because AE = 2\ BC$ $\therefore AE = 2 \times 8 = 16$ cm.

, $\because \overline{AE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ A) = m ($\angle$ B) = 90° (alternate angles)

$\therefore (ED)^2 = (AD)^2 + (AE)^2 = 144 + 256 = 400$

$\therefore ED = \sqrt{400} = 20$ cm. (The req.)

9

$\because$ The length $= \frac{48}{6} = 8$ cm.

$\therefore$ The length of diagonal $= \sqrt{8^2 + 6^2} = 10$ cm. (The req.)

10

ABFD is a rectangle

$\therefore DF = AB = 12$ cm.

, $AD = BE = 16$ cm.

$\therefore FC = 25 - 16 = 9$ cm.

, $\because$ m ($\angle$ DFC) = 90°

$\therefore (DC)^2 = (DF)^2 + (FC)^2 = 144 + 81 = 225$

$\therefore DC = \sqrt{225} = 15$ cm. (The req.)

11

Construction : Draw $\overline{DE} \perp \overline{BC}$

Proof : ABED is a rectangle

$\therefore DE = AB = 12$ cm.

, $AD = BE = 16$ cm.

$\therefore EC = 25 - 16 = 9$ cm.

, $\because$ m ($\angle$ DEC) = 90°

$\therefore (DC)^2 = (DE)^2 + (EC)^2 = 144 + 81 = 225$

$\therefore DC = \sqrt{225} = 15$ cm. (The req.)

Exercises

[A] : Choose The Correct Answer : -

1 In Δ ABC if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then m ($\angle$ A) = ...

(a) 180° (b) 45° (c) 90° (d) 120°

2 In Δ ABC , if m ($\angle$ A) + m ($\angle$ C) = m ($\angle$ B) , then m ($\angle$ B) = ...

(a) 180° (b) 90° (c) 45° (d) 360°

3 In Δ ABC , if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then m ($\angle$ C) .... 90°

(a) > (b) = (c) < (d) ≥

4 Δ ABC in which m ($\angle$ A) = 90° , then $(AC)^2 = (BC)^2$ .... $(AB)^2$

(a) + (b) × (c) – (d) ÷

5 If Δ ABC is right angled at B , then $(AB)^2$ = ...

(a) $(AC)^2 + (BC)^2$ (b) $(AC)^2 - (BC)^2$

(c) $(BC)^2 - (AC)^2$ (d) (AC) – (BC)

6 In Δ ABC if m ($\angle$ B) = 90° , AB = 6 cm. , BC = 8 cm. , then AC = ... cm.

(a) 100 (b) 8 (c) 6 (d) 10

7 In Δ ABC , if m ($\angle$ B) = 90° , AB = 3 cm. , BC = 4 cm. , then AC = ... cm.

(a) 7 (b) 1 (c) 5 (d) 4

8 In Δ ABC , if m ($\angle$ B) = 90° , AC = 10 cm. and BC = 8 cm.

, then AB = ... cm.

(a) 36 (b) $\sqrt{164}$ (c) 6 (d) 8

9 In Δ ABC , if m ($\angle$ B) = 90° , AB = 5 cm. , AC = 13 cm. , then BC = ... cm.

(a) 8 (b) 10 (c) 12 (d) 18

10 In Δ ABC , if m ($\angle$ A) = 90° , BC = 25 cm. and AC = 20 cm. , then AB = ... cm.

(a) 20 (b) 25 (c) 10 (d) 15

11 ABC is a right-angled triangle at B , if AC = 41 cm. and BC = 40 cm.

then AB = ...



(a) 11 cm. (b) 9 cm. (c) 36 cm. (d) 21 cm.

12 **In the opposite figure :**

AC = ... cm.

(a) 5 (b) 7

(c) 25 (d) 625

A, B, C; 3cm.; 4cm.

13 In the opposite figure :

m (∠ ACD) = ... °

(a) 40 (b) 140

(c) 90 (d) 50

14 In the opposite figure :

Δ ABC in which m (∠ B) = 90°

, AB = 3 cm. , BC = 4 cm.

, X , Y are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively , then :

(1) AC = ... cm.

(a) 7 (b) 6 (c) 5 (d) 4

(2) XY = ... cm.

(a) $\frac{5}{2}$ (b) 3 (c) 3.5 (d) 5

15 In the opposite figure :

The number of right-angled triangle is ....

(a) 3 (b) 2

(c) 1 (d) 0

16 In the opposite figure :

AC = ... cm.

(a) 5 (b) 7

(c) 8 (d) 6

17 The sum of measures of the angles of a triangle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

18 The sum of the interior angles of an isosceles triangle = ...

(a) 180° (b) 90° (c) 60° (d) 45°

19 The sum of the measures of the exterior angles of triangle = ...

(a) 90° (b) 180° (c) 360° (d) 120°

20 The measure of the exterior angle of the equilateral triangle = ...

(a) 60° (b) 90° (c) 30° (d) 120°

21 Any triangle has at least two .... interior angles.

(a) right (b) obtuse (c) acute (d) reflex

22 ✱ The triangle contains two .... angles at least

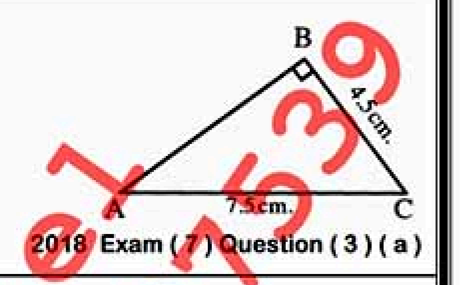
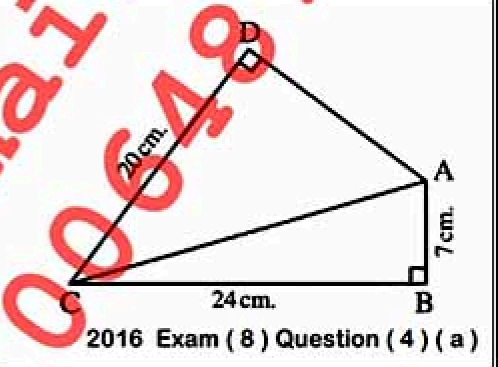
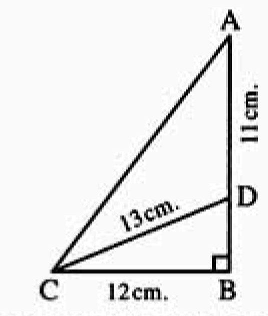
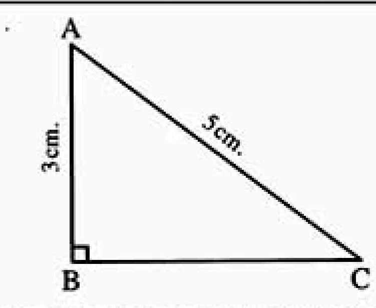
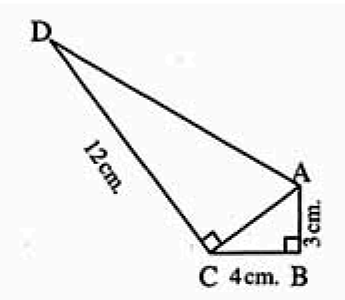
(a) acute (b) obtuse (c) right (d) reflex

[B] : Complete the Following : -

1	In Δ ABC , if m (∠ A) = 90° , then $(BC)^2$ = ...
2	In Δ ABC if m (∠ B) = 90° , then $(AC)^2$ = ... + ...
3	If ABC is a right-angled triangle at B , then $(BC)^2$ = ...
4	If ABC is right-angled triangle at B , then $(AB)^2 = (AC)^2$ – ....
5	In Δ XYZ , m (∠ Y) = 90° , then $(XZ)^2$ = ...
6	In Δ XYZ if m (∠ Y) = 90° , then $(XZ)^2$ = ...
7	In Δ ABC , if m (∠ A) = 90° , then $(BC)^2 = (AB)^2$ + ...
8	If $(AC)^2 = (AB)^2 - (BC)^2$, then the measure of angle (...............) = 90°
9	If XYZL is a rectangle , then $(XY)^2 + (YZ)^2$ = $(...............)^2$
10	In the rectangle ABCD , $(AB)^2 + (AD)^2$ = ...
11	If the measure of one angle of a triangle equals the sum of the measures of the other two angles , then the triangle is ....
12	Area of a triangle = ...
13	In the right-angled triangle , the area of the square on .... equals the sum of areas of the squares on the other two sides.
14	In the right-angled triangle , area of the square drawn on the hypotenuse equals .... of the lengths of the other two sides.
15	In the right-angled triangle , area of the square drawn on the hypotenuse equals ....
16	If Δ ABC is a right-angled triangle at A , then the longest side is called ....

17	If Δ ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then AC = ...
18	In Δ ABC , if m (∠ B) = 90° , AB = 3 cm. and BC = 4 cm. , then AC = ... cm.
19	The length of diagonal of a rectangle whose dimensions are 6 cm. and 8 cm. = ... cm.
20	If Δ XYZ is a right angled triangle at X , XY = 12 cm. and XZ = 9 cm. , then YZ = ...
21	If ABC is a right-angled triangle at B , AB = 20 cm. and AC = 25 cm. , then BC = ... cm.
22	**In the opposite figure :** AC = ... cm. (Figure: A, C, B; 10cm., 8cm.)
23	**In the opposite figure :** BC = ... cm. (Figure: A, B, C; 13cm., 5cm.)
24	**In the opposite figure :** AC = ... cm. (Figure: A, B, C; 5cm., 4cm.)
25	**In the opposite figure :** The value of x = ...° (Figure: 50°, x)
26	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ....
27	A circle its radius length 10 cm. , then its circumference = ... (Consider $\pi = 3.14$)

[C] : Essay Problems : -

1	**In the opposite figure :** ABC is a right-angled triangle at B , AC = 7.5 cm. , BC = 4.5 cm. **Find :** the length of $\overline{AB}$  2018 Exam (7) Question (3) (a)
2	**In the opposite figure :** AB = 7 cm. , CB = 24 cm. and DC = 20 cm. **Find :** (1) The length of $\overline{AC}$ (2) The length of $\overline{AD}$  2016 Exam (8) Question (4) (a)
3	**In the opposite figure :** Δ ABC in which m (∠ B) = 90° D ∈ $\overline{AB}$ such that AD = 11 cm. , if BC = 12 cm , DC = 13 cm. Find the length of each of : $\overline{AB}$, $\overline{AC}$  2018 Exam (10) Question (4) (a)
4	ABC is a right-angled triangle at B If AB = 3 cm. , AC = 5 cm. **Find :** the length of $\overline{BC}$  2018 Exam (15) Question (3) (b)
5	**In the opposite figure :** m (∠ B) = m (∠ ACD) = 90° , AB = 3 cm. , BC = 4 cm. , CD = 12 cm. **Find :** The length of each of $\overline{AC}$ and $\overline{AD}$  2017 Exam (7) Question (5) (a)

6

In the opposite figure :

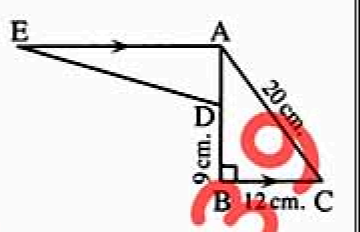
ΔABC in which $m(\angle B) = 90°$, $\overline{AE} \parallel \overline{BC}$

if BC = 12 cm. , AC = 20 cm. , $D \in \overline{AB}$

such that BD = 9 cm. , AE = 2 BC

Find :

① The length of $\overline{AD}$ ② The length of $\overline{ED}$



2018 Exam (13) Question (5) (a)



In the following figures , find the measure of the angle marked by (?)

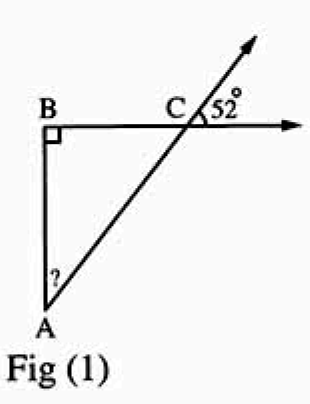


Fig (1)

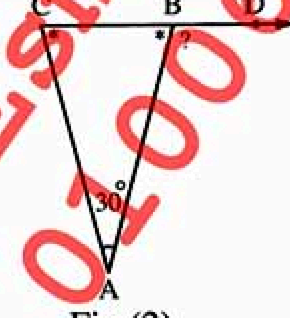


Fig (2)

2018 Exam (15) Question (5) (a)

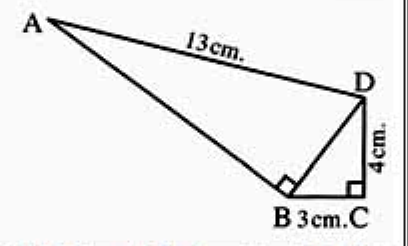
8

In the opposite figure :

BC = 3 cm. , CD = 4 cm. , DA = 13 cm.

$m(\angle ABD) = m(\angle C) = 90°$

Find with proof the length of : $\overline{BD}$, $\overline{AB}$



2018 Exam (6) Question (5) (b)

9

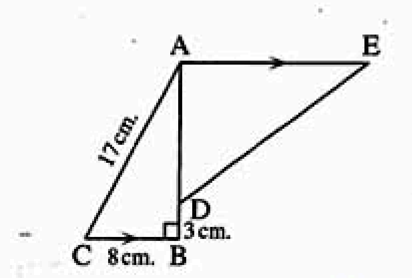
In the opposite figure :

ΔABC in which $m(\angle B) = 90°$, $\overline{AE} \parallel \overline{BC}$

, if BC = 8 cm. , AC = 17 cm.

, $D \in \overline{AB}$: BD = 3 cm. , AE = 2 BC

Find : The length of each of $\overline{AD}$ and $\overline{ED}$



2017 Exam (8) Question (3) (a)

10

Complete :

In the right-angled triangle the area of the square on the hypotenuse equals ....

2016 Exam (10) Question (5) (a)

11

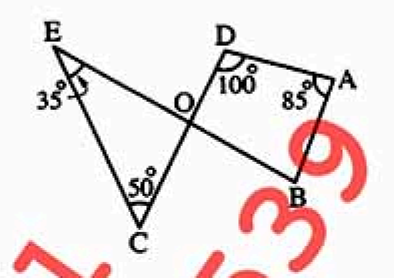
In the opposite figure :

$\overline{DC} \cap \overline{BE} = \{O\}$, m (∠ A) = 85° ,

m (∠ D) = 100° , m (∠ E) = 35° and

m (∠ C) = 50°

Find with proof : m (∠ B)

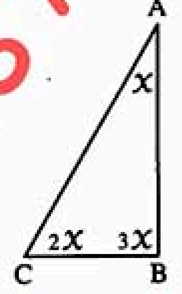


2016 Exam (2) Question (3) (a)

12

In the opposite proof :

Find with proof : the measures of the angles of Δ ABC



2016 Exam (15) Question (5) (b)

13

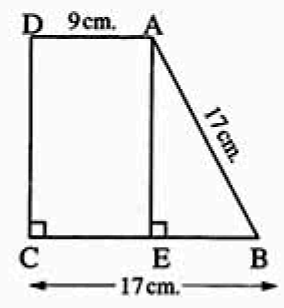
In the opposite figure :

ABCD is a trapezium , where $\overline{AD} // \overline{BC}$, m (∠ DCB) = 90°

, $\overline{AE} \perp \overline{BC}$ and AB = BC = 17 cm. , AD = 9 cm.

Find : ① The length of $\overline{DC}$

② The area of the trapezium ABCD



2017 Exam (15) Question (5) (a)

14

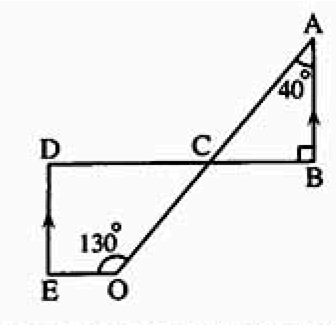
In the opposite figure :

$\overline{BD} \cap \overline{AO} = \{C\}$, $\overline{AB} // \overline{DE}$,

m (∠ A) = 40° , m (∠ B) = 90°

and m (∠ COE) = 130°

Find : m (∠ E)



2016 Exam (15) Question (4) (b)

Homework

[A] : Choose The Correct Answer : -

1 In Δ ABC if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then m ($\angle$ A) = ...

(a) 180° (b) 45° (c) 90° (d) 120°

2 If Δ ABC is right angled at B , then $(AB)^2$ = ...

(a) $(AC)^2 + (BC)^2$ (b) $(AC)^2 - (BC)^2$

(c) $(BC)^2 - (AC)^2$ (d) (AC) – (BC)

3 In Δ ABC , if m ($\angle$ A) = 90° , BC = 25 cm. and AC = 20 cm. , then AB = ... cm.

(a) 20 (b) 25 (c) 10 (d) 15

4 **In the opposite figure :**

Δ ABC in which m ($\angle$ B) = 90°

, AB = 3 cm. , BC = 4 cm.

, X , Y are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively , then :

(1) AC = ... cm.

(a) 7 (b) 6 (c) 5 (d) 4

(2) XY = ... cm.

(a) $\frac{5}{2}$ (b) 3 (c) 3.5 (d) 5

5 The measure of the exterior angle of the equilateral triangle = ...

(a) 60° (b) 90° (c) 30° (d) 120°

6 Any triangle has at least two .... angles.

(a) reflex (b) obtuse (c) acute (d) right

7 If X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ in Δ ABC and XY = 3 cm.

, then BC = ... cm.

(a) 3 (b) 5 (c) 6 (d) 9

8 Δ ABC in which m ($\angle$ A) = 90° , then $(AC)^2 = (BC)^2$ .... $(AB)^2$

(a) + (b) × (c) – (d) ÷

9 In Δ ABC , if m ($\angle$ B) = 90° , AB = 5 cm. , AC = 13 cm. , then BC = ... cm.

(a) 8 (b) 10 (c) 12 (d) 18

10 The sum of the measures of the exterior angles of triangle = ...

(a) 90° (b) 180° (c) 360° (d) 120°

11 The smallest number of the acute angle in any triangle is ....

(a) zero (b) 1 (c) 2 (d) 3

12 In ΔABC , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 8 cm. , then DE = ... cm.

(a) 16 (b) 8 (c) 4 (d) 2

13 In ΔABC , if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then m ($\angle$ C) .... 90°

(a) > (b) = (c) < (d) ≥

14 In ΔABC , if m ($\angle$ B) = 90° , AC = 10 cm. and BC = 8 cm. , then AB = ... cm.

(a) 36 (b) $\sqrt{164}$ (c) 6 (d) 8

15 **In the opposite figure :**

AC = ... cm.

(a) 5 (b) 7

(c) 25 (d) 625

C, 4cm., A, 3cm., B

16 The sum of the interior angles of an isosceles triangle = ...

(a) 180° (b) 90° (c) 60° (d) 45°

17 The right-angled triangle has .... right angle.

(a) 1 (b) 2 (c) 0 (d) 3

18 The line segment joining the midpoints of two sides of a triangle is .... the third side.

(a) perpendicular to (b) equal to (c) parallel to (d) bisect to

19 In ΔABC, if m ($\angle$ C) : m ($\angle$ A) : m ($\angle$ B) = 1 : 2 : 4 , then $\angle$ B is ....

(a) an obtuse (b) an acute (c) a right (d) otherwise

20 In ΔABC , if m ($\angle$ A) + m ($\angle$ C) = m ($\angle$ B) , then m ($\angle$ B) = ...

(a) 180° (b) 90° (c) 45° (d) 360°

21 In ΔABC , if m ($\angle$ B) = 90° , AB = 3 cm. , BC = 4 cm. , then AC = ... cm.

(a) 7 (b) 1 (c) 5 (d) 4

22 **In the opposite figure :**

AC = ... cm.

(a) 5 (b) 7

(c) 8 (d) 6

A, 3cm., C, 4cm., B

23 The sum of measures of the angles of a triangle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

24 ✱ The triangle contains two .... angles at least

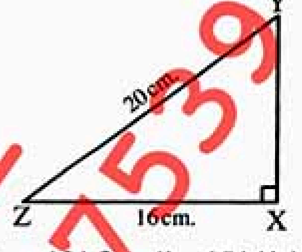
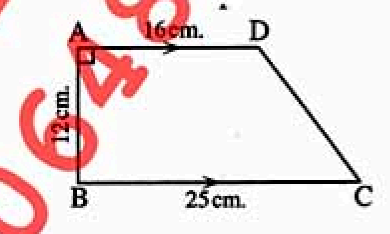
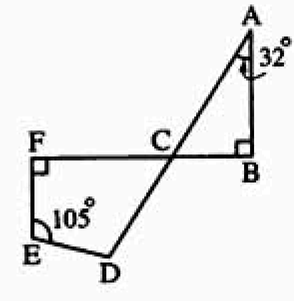
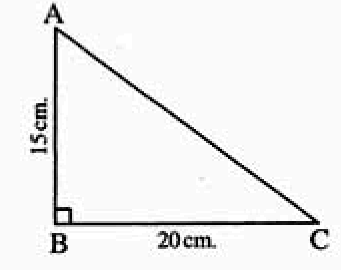
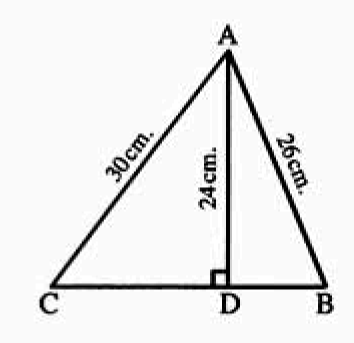
(a) acute (b) obtuse (c) right (d) reflex

[B] : Complete the Following : -

1	In Δ ABC , if m (∠ A) = 90° , then $(BC)^2$ = ...
2	If Δ XYZ is a right angled triangle at X , XY = 12 cm. and XZ = 9 cm. , then YZ = ...
3	If ABC is right-angled triangle at B , then $(AB)^2 = (AC)^2$ – ....
4	The sum of the measures of the angles of the quadrilateral equals ....
5	In the right-angled triangle , the area of the square on .... equals the sum of areas of the squares on the other two sides.
6	ABCD is parallelogram in which m (∠ A) = 100° , then m (∠ D) = ...°
7	In Δ ABC , if m (∠ B) = 90° , AB = 3 cm. and BC = 4 cm. , then AC = ... cm.
8	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ...°
9	If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ....
10	In the right-angled triangle , area of the square drawn on the hypotenuse equals ....
11	ABCD is a parallelogram in which m (∠ A) = 130° , then m (∠ B) = ...°
12	If $(AC)^2 = (AB)^2 - (BC)^2$, then the measure of angle (...............) = 90°
13	Each two opposite angles in a parallelogram are ....
14	Area of a triangle = ...
15	If ABCD is a parallelogram in which m (∠ A) = 80° , then m (∠ B) = ...
16	If ABC is a right-angled triangle at B , AB = 20 cm. and AC = 25 cm. , then BC = ... cm.

17	The measure of each interior angle of the regular pentagon = ...
18	In the rectangle ABCD , $(AB)^2 + (AD)^2 =$ ....
19	ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ...
20	The length of diagonal of a rectangle whose dimensions are 6 cm. and 8 cm. = ... cm.
21	If ABC is a right-angled triangle at B , then $(BC)^2 =$ ....
22	The sum of the measures of the exterior angles of the convex polygon = ...
23	In Δ ABC , if m (∠ A) = 90° , then $(BC)^2 = (AB)^2 +$ ....
24	If two opposite sides in the quadrilateral are parallel , then it is called ....
25	If Δ ABC is a right-angled triangle at A , then the longest side is called ....
26	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{3}$ m (∠ Y) , then m (∠ L) = ...°
27	In Δ XYZ , m (∠ Y) = 90° , then $(XZ)^2 =$ ....
28	The measure of each interior angle of the regular hexagon is ....°
29	In the right-angled triangle , area of the square drawn on the hypotenuse equals .... of the lengths of the other two sides.
30	If ABCD is a parallelogram in which : m (∠ A) = 120° , then m (∠ B) = ...°
31	In Δ ABC if m (∠ B) = 90° , then $(AC)^2 =$ .... + ...
32	A circle its radius length 10 cm. , then its circumference = ... (Consider π = 3.14)
33	If the measure of one angle of a triangle equals the sum of the measures of the other two angles , then the triangle is ....
34	ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...

[C] : Essay Problems : -

1	**In the opposite figure :** XYZ is right-angled triangle at X , YZ = 20 cm. , XZ = 16 cm. **Find :** the length of $\overline{XY}$  2018 Exam (2) Question (5) (b)
2	**In the opposite figure :** ABCD is a quadrilateral in which m (∠ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm. and $\overline{AD}$ // $\overline{BC}$ **Find with proof :** The length of $\overline{DC}$  2017 Exam (10) Question (5) (b)
3	**In the opposite figure :** $\overline{AD} \cap \overline{FB} = \{C\}$, m (∠ A) = 32° , m (∠ B) = m (∠ F) = 90° and m (∠ E) = 105° **Find :** m (∠ D)  2016 Exam (7) Question (4) (b)
4	**In the opposite figure :** Δ ABC in which m (∠ B) = 90° , AB = 15 cm. , BC = 20 cm. **Find with proof :** The length of $\overline{AC}$  2017 Exam (5) Question (4) (a)
5	**In the opposite figure :** $\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm. **Find :** (1) The length of $\overline{BC}$ (2) The area of Δ ABC  2018 Exam (9) Question (4) (a)

6	**In the opposite figure :** $\overline{AB} \parallel \overline{DE}$, m (∠ B) = 90° , m (∠ A) = 30° and m (∠ O) = 130° **Find with proof :** m (∠ E) 2016 Exam (8) Question (3) (b)
7	**In the opposite figure :** m (∠ B) = 90° , AB = 12 cm. and AC = 15 cm. **Find :** the length of $\overline{BC}$ 2016 Exam (6) Question (4) (a)
8	**In the opposite figure :** **Prove that :** ① BC = 14 cm. ② Area of Δ ABC = 84 cm². 2018 Exam (1) Question (5) (a)
9	Find the length of the diagonal of a rectangle whose area 48 cm². and of width 6 cm. 2018 Exam (4) Question (4) (a)
10	**In the opposite figure :** $\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$, m (∠ A) = 30° , m (∠ EOC) = 120° , **Find :** m (∠ E) Model 2018 Exam (2) Question (4) (b)
11	**In the opposite figure :** ABC is a triangle in which m (∠ B) = 90° , AB = 9 cm. , BC = 12 cm. **Find with proof :** The length of $\overline{AC}$ 2017 Exam (7) Question (3) (a)

12	**In the opposite figure :** m (∠ A) = m (∠ C) = 90° , AB = 20 cm. , CD = 24 cm. , AD = 15 cm. **Find :** the perimeter of Δ BCD 2018 Exam (3) Question (3) (b)
13	Find the length of the diagonal of a rectangle whose area 48 cm^2 and of width 6 cm. 2017 Exam (10) Question (3) (b)
14	**In the opposite figure :** $\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$, m (∠ A) = 20° and m (∠ EOC) = 110° **Find :** m (∠ E) 2016 Exam (9) Question (3) (a)
15	**In the opposite figure :** ABC is a triangle in which m (∠ B) = 90° , AB = 6 cm. , AC = 10 cm. **Find with proof :** the length of $\overline{BC}$ 2018 Exam (8) Question (3) (a)
16	**In the opposite figure :** m (∠ B) = m (∠ ACD) = 90° AB = 9 cm. , BC = 12 cm. , CD = 20 cm. **Find length of :** $\overline{AC}$ and $\overline{AD}$ 2018 Exam (5) Question (4) (b)
17	If the area of a rectangle equals 12 cm^2 and its length is 4 cm. , then find the length of its diagonal. 2016 Exam (1) Question (5) (a)
18	ABCD is a rectangle in which AB = 6 cm. , AC = 10 cm. **Find :** The length of $\overline{AD}$ 2017 Exam (9) Question (3) (b)

Prep [1]

Geometry - Second Term

Unit [3] - Part [5]



Mr. Mahmoud Esmaiel

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الاســم ....

Lesson [5] : Geometric Transformation

Prelude

Notice the changing which happened to the position of the letter E corresponding to its previous position directly in each case of the following cases :

The first case	E ➜ Ǝ ➜ E ➜ Ǝ
The second case	E ➜ E ➜ E ➜ E
The third case	•E ➜ m ➜ Ǝ ➜ Ɯ •E ➜ Ɯ ➜ Ǝ ➜ m

- The geometric transformation in the first case is called : **reflection**
- The geometric transformation in the second case is called : **translation**
- The geometric transformation in the third case is called : **rotation**

The concept of the geometric transformation

In each of the following figures , notice the image of Δ ABC and deduce what happened to it :

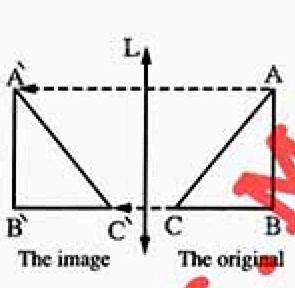


Fig. (1)

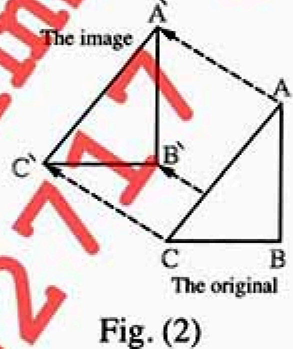


Fig. (2)

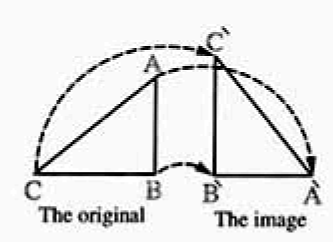


Fig. (3)

In each of the previous figures , notice that :

- The point A has been transferred to A`
- The point B has been transferred to B`
- The point C has been transferred to C`

Thus , all the points of Δ ABC have been transferred to another position , then we say that Δ ABC has been transformed from position to another position.

Geometric transformations have many types as : reflection , translation and rotation which we will study each of them in details in the following lessons.

Lesson [6] : Reflection

Definition of reflection in a straight line

Reflection in the straight line L maps each point A to the point À in the same plane such that :

1 If $A \notin L$, then the straight line L is the perpendicular bisector to the line segment $\overline{A\grave{A}}$

2 If $B \in L$, then B is reflected onto itself

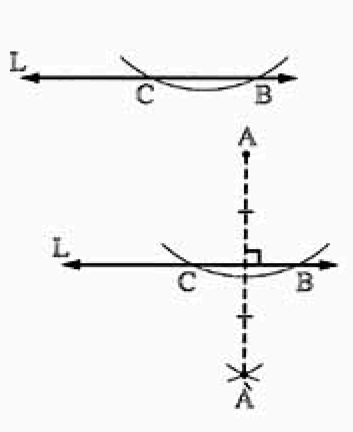
i.e. B̀ coincides B

Finding the image of a point by reflection in a given straight line

• To find À which is the image of A by reflection in the straight line L , we do as follows :

1 Draw an arc of a circle with centre A to cut L at B and C

2 With the same radius length taking B and C as centres , draw two arcs in the other side of the straight line L to intersect at À , then À is the image of A by reflection in L



Check by measuring that :

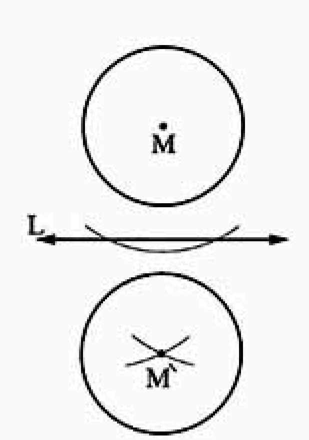
$\overline{A\grave{A}} \perp L$ and L bisects $\overline{A\grave{A}}$

Finding the image of a circle by reflection in a given straight line

• To find the image of a circle M by reflection in the straight line L , we do as follows :

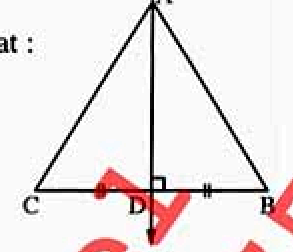
1 Find the image of the centre M by reflection in L as we did before , say M̀

2 Use the compasses with radius length equal to the radius length of the circle M to draw a circle with centre M̀ that will be the image of the circle M by reflection in L



Symmetry

- **In the opposite figure :**



ABC is a triangle , $\overleftrightarrow{AD} \perp \overline{BC}$, D is the midpoint of $\overline{BC}$, we find that :

- The image of A by reflection in L is itself (A)
- The image of B by reflection in L is C
- The image of C by reflection in L is B



i.e. The image of Δ ABC by reflection in L is Δ ACB

We can say that Δ ABC is transformed to itself by reflection in the straight line L ,

Therefore the straight line L is called the axis of symmetry of Δ ABC

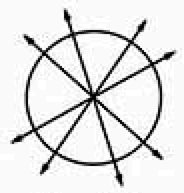
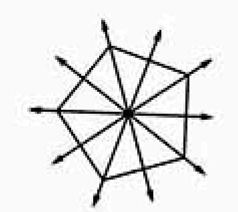
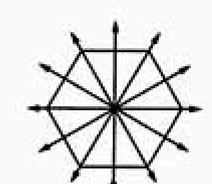
From the previous , we can deduce the definition of the axis of symmetry as follows :

If the reflection in a line transforms the figure to itself , then this line is called an axis of symmetry of the figure.

The axis of symmetry divides the figure into two congruent figures.

The axes of symmetry of some geometric figures

The figure	An isosceles triangle	An equilateral triangle	Scalene
Number of axes of symmetry	1	3	Zero (does not exist)
The figure	Parallelogram	Rectangle	Rhombus
Number of axes of symmetry	Zero (does not exist)	2	2
The figure	Square	Trapezium	An isosceles trapezium
Number of axes of symmetry	4	Zero (does not exist)	1

The figure	 The circle	 The regular pentagon	 The regular hexagon
Number of axes of symmetry	An infinite number	5	6

Definition of reflection in a point

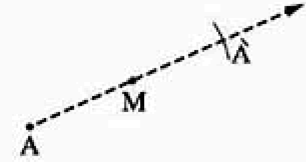


Reflection in a point M maps each point A in the plane to the point À in the same plane where M is the midpoint of the line segment $\overline{AA'}$, the point M is called the centre of reflection and the image of M by reflection in M is itself.

Finding the image of a point by reflection in a given point

- **To find the image of a point as A by reflection in M , we do as follows :**

1. Draw $\overrightarrow{AM}$
2. Using the compasses with open length equals MA , then use M as a centre and draw an arc to intersect $\overrightarrow{AM}$ at a point as À , then À is the image of the point A by reflection in the point M



3. From the previous , we found that : MA = MÀ

Lesson [7] : Translation

i.e. **To determine the new position of the car after its movement , we should know two important elements which are :**

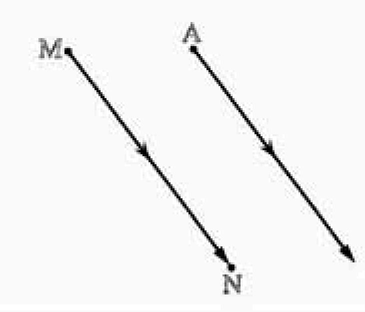
1. The magnitude of the translation (25 metres).
2. The direction of the translation (forward in a straight line).

According to this, we can say that :

Translation is a geometrical transformation which maps each point A in the plane to another point À in the same plane with a constant distance in a certain direction.

Translation in the plane

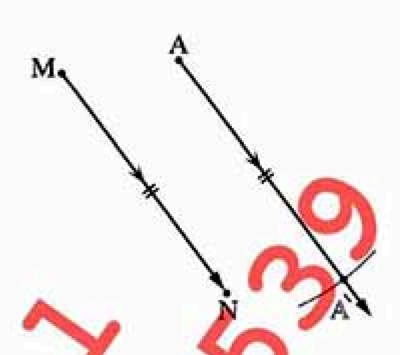
Finding the image of a point by a given translation



- **To find À which is the image of A by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

1. Draw from A a ray parallel to $\overrightarrow{MN}$ and in the same direction.

2 By the compasses in A as a centre with radius = MN , draw an arc to intersect the ray drawn from A at the point À (AÀ = MN and $\overline{AÀ} // \overline{MN}$)



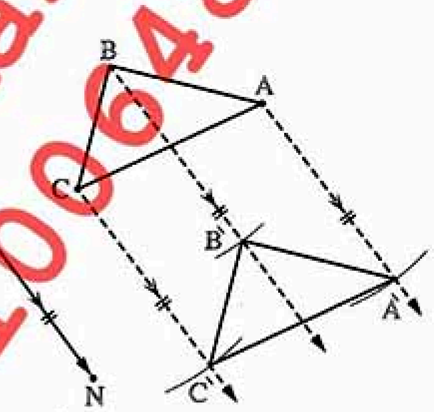
- **Then À is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$**

Finding the image of a polygon by a given translation

- **To find the image of a polygon as Δ ABC by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

1 Find the image of each vertex of the vertices of Δ ABC by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before (say À for A , B̀ for B and C̀ for C)

2 Draw $\overline{ÀB̀}$, $\overline{B̀C̀}$ and $\overline{C̀À}$ then Δ ÀB̀C̀ is the image of Δ ABC by translation MN in the direction of $\overrightarrow{MN}$



Check that :

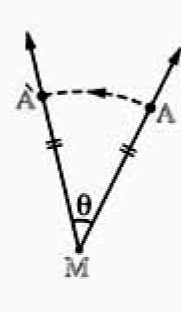
- AB = ÀB̀ , BC = B̀C̀ and CA = C̀À
- m (∠ A) = m (∠ À) , m (∠ B) = m (∠ B̀) , m (∠ C) = m (∠ C̀)

From the previous , we deduce that translation is a geometrical transformation which maps the geometrical figure to another geometrical figure congruent to it.

Lesson [8] : Rotation

The concept of rotation

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point À in the same plane such that m (∠ AMÀ) = θ , MA = MÀ It is denoted by R (M , θ) where :



- M is the centre of rotation.
- θ is the measure of the angle of rotation.

According to this concept , the rotation is determined completely if we know the following elements

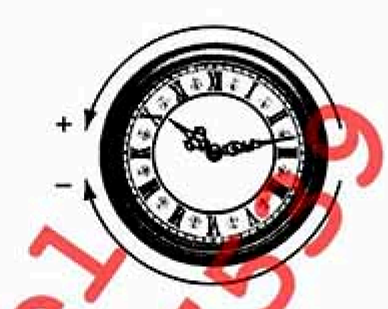
1 The centre of rotation.

2 The measure of the angle of rotation (θ)

3 The direction of rotation.

Remark [1]

The measure of rotation angle is positive
if the rotation is anticlockwise and it is negative
if the rotation is clockwise.

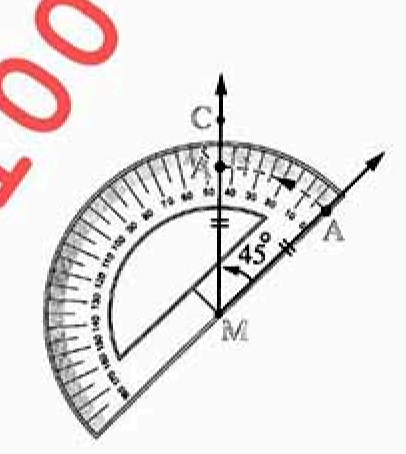


Rotation in the plane

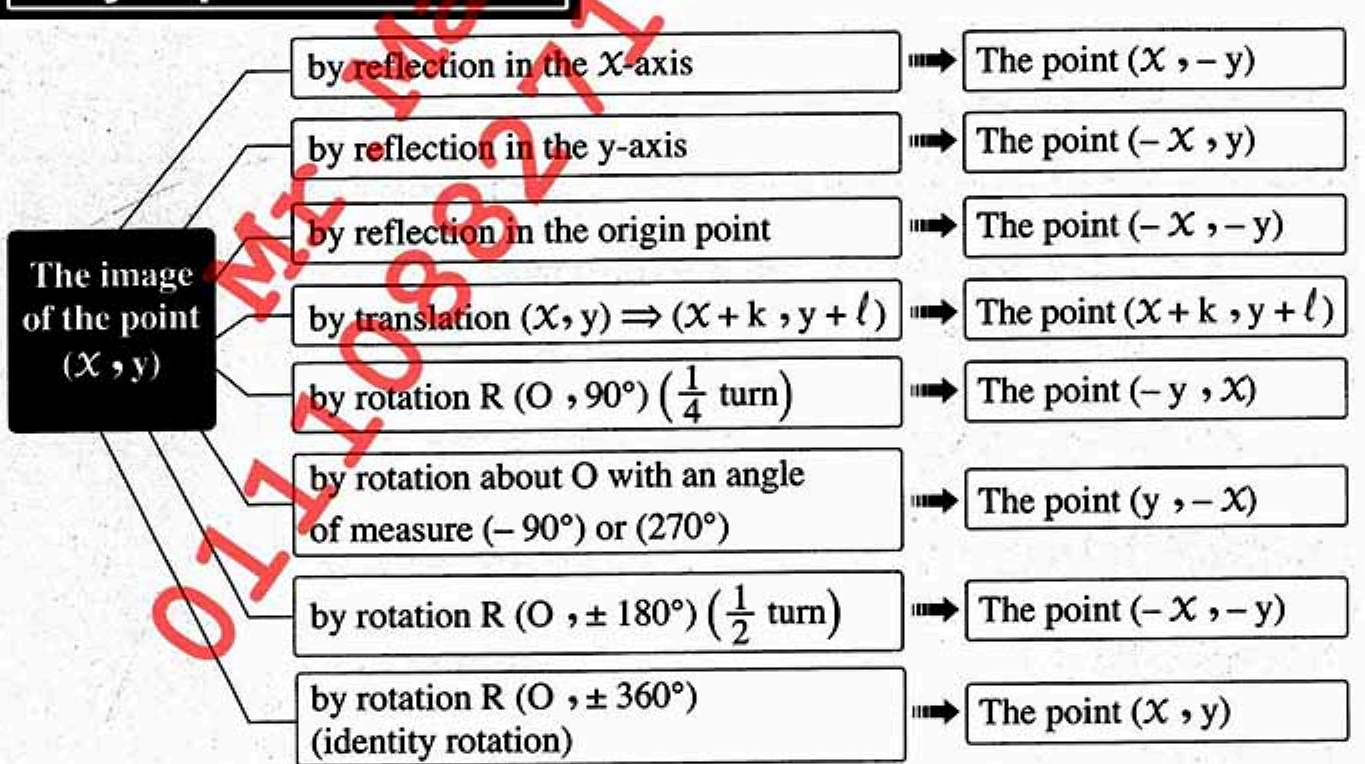
Finding the image of a given point by a given rotation

Firstly : Finding the image of the point A by rotation around the point M with an angle of measure 45° i.e. R (M , 45°) :

- Draw the ray $\overrightarrow{MA}$
- Put the protractor with its straight edge on $\overrightarrow{MA}$ and in the anticlockwise direction , then draw $\overrightarrow{MC}$ such that m (∠ AMC) = 45°
- Use the compasses at the point M as a centre with radius = MA , draw an arc to cut $\overrightarrow{MC}$ at À then À is the image of the point A by rotation around M with an angle of measure 45°

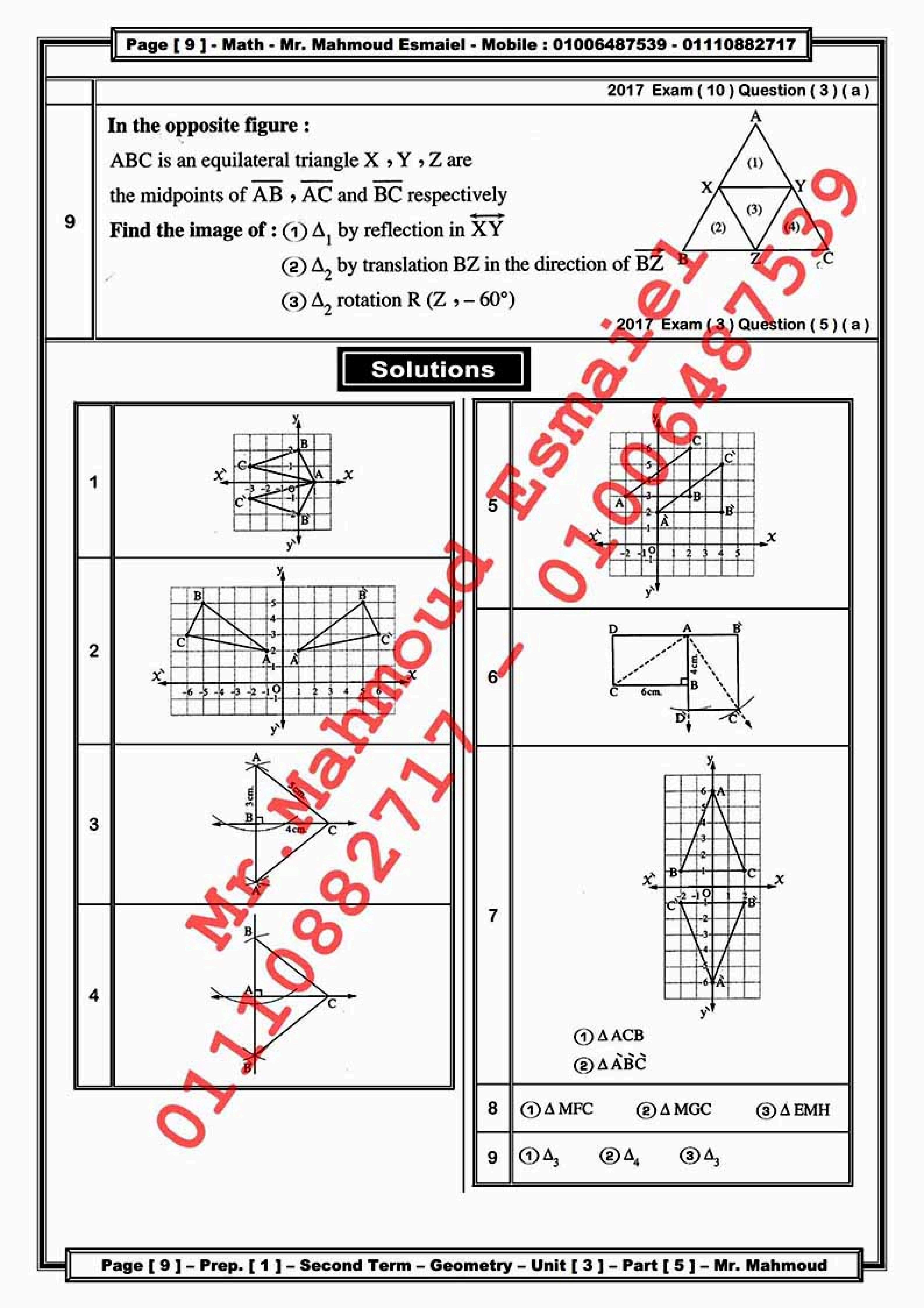


Very Important Notes :



Examples :

1	Using the lattice , draw Δ ABC where A (1 , 0) , B (0 , 2) and C (– 3 , 1) , then draw its image by reflection in x-axis. **2017 Exam (1) Question (5) (b)**
2	Using the lattice , find the image of the triangle ABC by reflection in y-axis where A (– 1 , 2) , B (– 5 , 5) , C (– 6 , 3) **2017 Exam (12) Question (3) (b)**
3	Draw the triangle ABC in which AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then draw its image by reflection in $\overleftrightarrow{BC}$ **2017 Exam (8) Question (5) (b)**
4	Draw Δ ABC in which AB = 3.5 cm. , m ($\angle$ A) = 90° , AC = 5 cm. , then draw its image by reflection in $\overleftrightarrow{AC}$ **2017 Exam (9) Question (4) (a)**
5	Using the square lattice , draw Δ ABC where A (– 2 , 3) , B (2 , 3) , C (2 , 6) , then find the image of Δ ABC by translation $(x , y) \longrightarrow (x + 2 , y - 1)$ **2017 Exam (13) Question (5) (b)**
6	Draw the rectangle ABCD in which BC = 6 cm. and AB = 4 cm. , draw the image of the rectangle ABCD by rotation R (A , 90°) **2016 Exam (5) Question (5) (a)**
7	In square lattice , draw Δ ABC where A (0 , 6) , B (– 2 , 1) , C (2 , 1) **, then find :** (1) The image of Δ ABC by reflection in y-axis. (2) The image of Δ ABC by rotation (0 , 180°) **2017 Exam (6) Question (4) (b)**
8	**In the opposite figure :** ABCD is a square in which E , F , G and H are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ respectively **Find :** (1) The image of Δ MGD by rotation at point M by angle with measure (– 90°) (2) The image of Δ MGD by reflection in $\overleftrightarrow{EG}$ (3) The image of Δ MGD by translation (DH) in direction $\overrightarrow{DH}$



2017 Exam (10) Question (3) (a)

9

In the opposite figure :

ABC is an equilateral triangle X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively

Find the image of : ① Δ_1 by reflection in $\overleftrightarrow{XY}$

② Δ_2 by translation BZ in the direction of $\overrightarrow{BZ}$

③ Δ_2 rotation R (Z , – 60°)

2017 Exam (3) Question (5) (a)

Solutions

No.	Solution
1	(figure)
2	(figure)
3	(figure)
4	(figure)
5	(figure)
6	(figure)
7	① Δ ACB ② Δ ÀB̀C̀
8	① Δ MFC ② Δ MGC ③ Δ EMH
9	① Δ_3 ② Δ_4 ③ Δ_3

Exercises

[A] : Choose The Correct Answer : -

1 In the opposite figure :

The image of the point A by reflection in the point M is ……………

(a) A (b) B

(c) C (d) D

2 The image of the point (2 , – 5) by reflection in X-axis is ……………

(a) (– 2 , 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

3 The image of the point (3 , – 5) by reflection in the y-axis is ……………

(a) (3 , 5) (b) (– 3 , – 5) (c) (5 , – 3) (d) (– 3 , 5)

4 The image of the point (– 1 , 4) by the translation (3 , – 2) is ……………

(a) (2 , 2) (b) (– 2 , – 2) (c) (2 , – 2) (d) (– 2 , 2)

5 If the image of the point (3 , – 2) by rotation about the origin point is itself , then the measure of the rotation angle is ……………

(a) 90° (b) 360° (c) 180° (d) 270°

6 The image of the point (– 5 , 7) by rotation about the origin point with an angle of measure 90° is ……………

(a) (– 7 , – 5) (b) (5 , – 7) (c) (– 7 , 5) (d) (7 , 5)

7 The image of the point (2 , – 1) by reflection in the X-axis is ……………

(a) (2 , 1) (b) (1 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)

8 The image of the point (2 , 5) by reflection in y-axis is ……………

(a) (– 2 , 5) (b) (5 , 2) (c) (– 2 , – 5) (d) (2 , – 5)

9 If the image of the point (– 1 , 3) by a translation is (1 , 4) , then the image of the point (3 , – 2) by the same translation is ……………

(a) (1 , 2) (b) (5 , – 1) (c) (– 1 , 2) (d) (– 2 , – 1)

10 If $\acute{A}$ (4 , – 5) is the image of A by translation (X , y) ⟶ (X – 2 , y + 1) then the point A = ……………

(a) (6 , – 4) (b) (4 , – 4) (c) (2 , – 4) (d) (6 , – 6)

11 The image of the point (4 , 3) by rotation about the origin point with an angle of measure 90° is ……………

(a) (4 , 3) (b) (– 4 , 3) (c) (– 3 , 4) (d) (– 3 , – 4)

12	The image of the point (1 , 5) by reflection in the x-axis is .... (a) (1 , 5) (b) (1 , – 5) (c) (5 , 1) (d) (– 5 , 1)	
13	The image of the point (2 , – 1) by reflection in y-axis is .... (a) (2 , 1) (b) (– 2 , – 1) (c) (– 2 , 1) (d) (2 , – 1)	
14	The image of the point (– 1 , 3) by translation (4 , – 2) is .... (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)	
15	If the point (a , – 1) is the image of (2 , 4) by translation $(x , y) \longrightarrow (x + 1 , y - b)$, then (a , b) is .... (a) (3 , 3) (b) (1 , 3) (c) (3 , 5) (d) (1 , – 5)	
16	The image of the point (3 , – 5) by rotation R (0 , 90°) is .... (a) (– 3 , 5) (b) (– 3 , – 5) (c) (5 , 3) (d) (5 , – 3)	
17	The image of the point (– 1 , 3) by reflection in x-axis is .... (a) (1 , 3) (b) (3 , – 1) (c) (– 1 , – 3) (d) (1 , – 3)	
18	The image of the point (1 , – 4) by reflection in y-axis is .... (a) (– 1 , – 4) (b) (4 , 1) (c) (– 1 , 4) (d) (– 4 , – 1)	
19	The image of the point (– 1 , 3) by translation $(x + 4 , y - 2)$ is .... (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)	
20	The image of the point (5 , – 3) by translation 3 units in negative direction of x-axis is .... (a) (– 3 , 5) (b) (– 2 , – 3) (c) (2 , – 3) (d) (5 , 0)	
21	The image of the point (2 , 3) by rotation about the origin point with an angle of measure 90° is .... (a) (2 , – 3) (b) (– 2 , 3) (c) (– 3 , 2) (d) (2 , 3)	
22	The reflection in the x-axis maps the point B (x , y) to the point B̀ (............... , ...) (a) (x , y) (b) $(x , -y)$ (c) $(-x , -y)$ (d) $(-x , y)$	
23	The image of the point (– 1 , 3) by reflection in y-axis is .... (a) (1 , 3) (b) (3 , – 1) (c) (– 1 , – 3) (d) (1 , – 3)	
24	The image of the point (– 1 , 2) by translation of magnitude 3 units in the positive direction of y-axis is .... (a) (– 2 , 2) (b) (– 1 , 5) (c) (– 1 , – 5) (d) (– 2 , 5)	
25	The image of the point (5 , – 3) by translation (1 , 1) is .... (a) (6 , – 4) (b) (4 , – 2) (c) (6 , – 2) (d) (4 , 2)	

[B] : Complete the Following : -

1	The image of the point (4 , 1) by reflection in the origin point is ....
2	The reflection in the x-axis maps the point B (4 , 2) to the point $\grave{B}$ (.......... , ...)
3	The translation is determined by .... and ....
4	The image of the point (4 , 6) by translation : $(x , y) \longrightarrow (x , y - 7)$ is ....
5	The image of the point (x , y) by rotation about the origin point is itself , then the measure of rotation angle is ....
6	The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ....
7	The point (3 , 7) is the image of the point .... by reflection in the x-axis.
8	The reflection in a line reserves .... , ...
9	The image of the point (– 4 , 5) by translation (2 , – 3) is ....
10	The neutral rotation maps the figure to ....
11	The image of the point (2 , – 3) by rotation about the origin point with an angle of measure 180° is ....
12	The image of the point (3 , 2) by reflection in the x-axis is ....
13	(– 3 , 2) is the image of the point (3 , 2) by reflection in .... aixs.
14	The image of the point (4 , – 5) by translation (– 2 , 4) is ....
15	In the rotation around O with an angle of measure ....° , the image of each point coincides exactly with itself.
16	(– 3 , 2) is the image of the point (– 3 , – 2) by reflection in ....

17	The image of the point (3 , – 5) by translation 3 units in the positive direction of x-axis is ....
18	Rotation in the plane reserves .... of angles.
19	The image of the point (3 , – 5) by rotation R (O , 90°) is ....
20	**In the opposite figure :** ABCD is a square , X , Y , Z , L are midpoints of its sides then : (1) The image of Δ AXO by reflection in $\overleftrightarrow{XZ}$ is .... (2) The image of Δ DZO by rotation R (O , 90°) is .... (3) The image of Δ LOD by translation of magnitude DO in the direction $\overrightarrow{DO}$ is ....
21	The image of the point (2 , – 3) by reflection in x-axis is ....
22	The image of the point (3 , – 5) by reflection in the y-axis is ....
23	The image of the point (3 , 5) by translation : $(x , y) \longrightarrow (x + 3 , y - 1)$ is ....
24	**In the opposite figure :** The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ....
25	The image of the point (3 , 2) by rotation with an angle of measure 90° about the origin point is ....
26	The image of the point C (x , y) is the point C̀ $(- x , - y)$ by rotation around the origin point O and an angle whose measure is ....°
27	The image of the point (2 , 1) by reflection in x-axis is ....
28	The image of the point (– 3 , 5) by reflection in the y-axis is ....

[C] : Essay Problems : -

1	If the image of the point A by reflection in the x-axis is (2 , 3) , locate the point A , then draw the image of A by reflection in the y-axis 2016 Exam (1) Question (4) (a)
2	On the orthogonal square lattice draw the triangle ABC where : A = (1 , 1) , B = (3 , 4) and C = (5 , 2) , then draw the image of the triangle ABC by rotation about the origin point with an angle of measure 90° 2016 Exam (7) Question (5) (a)
3	On the square lattice , draw $\overline{AB}$ where A (3 , 2) , B (– 1 , 1) then find its image by translation (– 2 , – 5) 2017 Exam (7) Question (3) (b)
4	Draw Δ ABC in which : AB = 5 cm. , BC = 4 cm. and m (∠ B) = 90° , then find the image of Δ ABC by reflection on $\overleftrightarrow{AB}$ 2016 Exam (6) Question (3) (b)
5	Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis. Model 2018 Exam (2) Question (4) (a)
6	Draw the rectangle ABCD in which BC = 6 cm. and AB = 4 cm. , draw the image of the rectangle ABCD by rotation R (A , 90°) 2016 Exam (5) Question (5) (a)
7	On a square lattice , draw $\overline{AB}$ where A (2 , 3) and B (4 , 1) , then draw the image of $\overline{AB}$ By rotation about the origin point with an angle of measure : (1) 90° (2) 180° 2017 Exam (5) Question (5) (a)
8	Using the square lattice , draw $\overline{AB}$ where A (4 , 2) and B (– 1 , – 1) , then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$ 2017 Exam (14) Question (3) (b)
9	Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3) , **then draw its image :** (1) By reflection in y-axis. (2) By rotation about origin point with an angle of measure 180° 2018 Exam (10) Question (5) (a)
10	Draw the image of Δ ABC in which : AB = 6 cm. , BC = 4 cm. , AC = 5 cm. , by reflection in $\overleftrightarrow{AC}$ 2018 Exam (12) Question (4) (b)
11	On the square lattice , draw Δ ABC where A (1 , 1) , B (5 , 2) , C (3 , 5) , then find its image by reflection in x-axis. 2017 Exam (7) Question (4) (b)

12

In the opposite figure :

ABC is an equilateral triangle X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively

Find the image of : (1) Δ_1 by reflection in $\overleftrightarrow{XY}$

(2) Δ_2 by translation BZ in the direction of $\overrightarrow{BZ}$

(3) Δ_2 rotation R (Z , – 60°)

2017 Exam (3) Question (5) (a)

13

Draw the rectangle ABCD on a square lattice where A (0 , 0) , B (0 , 2) , C (4 , 2) , D (4 , 0) , then find its image by rotation about the origin point with an angle of measure 180°

2017 Exam (3) Question (4) (a)

14

On a square lattice , draw Δ ABC , such that : A (4 , 4) , B (4 , 2) and C (1 , 2) Find its image by translation (x – 2 , y + 1)

2016 Exam (8) Question (5) (b)

15

In the opposite figure :

Draw the image of the Δ ABC by the reflection in the x-axis where A (2 , 2) , C (6 , 4) and B (6 , 6)

2018 Exam (1) Question (4) (a)

16

Using the lattice , find the image of the triangle ABC by reflection in y-axis where A (– 1 , 2) , B (– 5 , 5) , C (– 6 , 3)

2017 Exam (12) Question (3) (a)

17

In square lattice , draw Δ ABC where A (0 , 6) , B (– 2 , 1) , C (2 , 1)

, then find :

(1) The image of Δ ABC by reflection in y-axis.

(2) The image of Δ ABC by rotation (0 , 180°)

2017 Exam (6) Question (4) (b)

18

On square lattice , draw the triangle whose vertices are A (5 , 5) , B (5 , 3) , C (2 , 3) , then determine each of the following :

(1) The image of Δ ABC by translation (– 2 , 2)

(2) The image of Δ ABC by reflection in y-axis.

2017 Exam (11) Question (5) (a)

19	If the point A is the image of point B $(-1, 4)$ by refection in x-axis Find the image of A by translation $(1, 4)$ 2017 Exam (11) Question (4) (b)
20	In the square net draw Δ ABC in which A $(-2, 2)$, B $(3, 1)$, and C $(2, 5)$, then find its image by reflection in the origin point. 2018 Exam (6) Question (5) (a)
21	**In the opposite figure :** ABCD is a square in which E , F , G and H are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ respectively **Find :** ① The image of Δ MGD by rotation at point M by angle with measure $(-90°)$ ② The image of Δ MGD by reflection in $\overleftrightarrow{EG}$ ③ The image of Δ MGD by translation (DH) in direction $\overrightarrow{DH}$ 2017 Exam (10) Question (3) (a)
22	If the point $\grave{A}$ $(4, 5)$ is the image of the point A $(1, 7)$ by translation $(x, y) \longrightarrow (x + a, y + b)$ **find :** ① The value of a and b ② The image of the point B $(2, -4)$ by the same translation. 2018 Exam (12) Question (5) (b)
23	**Complete :** If the point $(1, 4)$ is the image of the point $(-1, 3)$ by a translation (x, y) , then the image of the point $(3, -2)$ by the same translation is .... 2018 Exam (10) Question (4) (b)
24	**In the opposite figure :** ABCD is a square , whose diagonals intersect at M **Find :** ① The image of Δ ABC by reflection in $\overleftrightarrow{AC}$ ② The image of Δ MAB by rotation about M with angle of measure $(-90°)$ 2018 Exam (13) Question (5) (b)
25	On the lattice , find the image of the triangle LMN , where L $(-4, -1)$, M $(-1, -3)$, N $(0, -1)$ by reflection on the x-axis 2016 Exam (5) Question (3) (b)
26	Find the image of the point $(2, -1)$ by rotation R $(O, 180°)$ 2016 Exam (4) Question (5) (a)

27

Find $\overline{A'B'}$ the translated image of $\overline{AB}$, where A (2 , 1) and B (2 , 4) when translated MN units in the direction of $\overrightarrow{MN}$, where M (– 2 , 5) and N (3 , 7)

2018 Exam (1) Question (3) (a)

28

In the opposite figure :

ABCD is a square of side length 6 cm.

and the origin point is its centre.

Find :

(1) The image of Δ AOM by translation 3 cm. in the direction of $\overrightarrow{AB}$

(2) The image of Δ AOM by rotation R (O , 90°)

(3) The image of Δ AOM by reflection in $\overleftrightarrow{EN}$

2018 Exam (8) Question (4) (a)

29

Using the square lattice , draw the triangle ABC where A (3 , – 1) , B (5 , 2) and C (– 2 , 4) , then draw its image by rotation R (O , 180°)

2018 Exam (9) Question (3) (b)

30

In the opposite figure :

ABED is a trapezium in which $\overline{AB}$ // $\overline{DE}$

, m (∠ D) = 90° , $\overline{BC} \perp \overline{DE}$

, BE = DE = 17 cm. , AB = 9 cm.

Find : (1) The length of $\overline{AD}$

(2) The image of $\overline{AB}$ by translation of magnitude AD in direction of $\overrightarrow{AD}$

2017 Exam (13) Question (4) (a)

31

Draw Δ ABC and draw its image by reflection in y-axis where :

A (– 6 , – 1) , B (– 2 , – 1) , C (– 5 , – 6)

2017 Exam (4) Question (5) (b)

32

In the opposite figure :

ABCD is a square , whose diagonal intersect at M , **find :**

(1) The image of Δ ABC by reflection in $\overleftrightarrow{AC}$

(2) The image of Δ MAB by reflection in point M

2018 Exam (10) Question (3) (b)

33

On a square lattice , draw the triangle ABC where A (4 , 4) , B (4 , 2) , C (1 , 2) , then find its image by rotation about the origin point with an angle of measure 180°

2018 Exam (15) Question (5) (b)

Homework

[A] : Choose The Correct Answer : -

1 The image of the square by rotation about the origin point with an angle of measure 90° is ……………
(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 The point whose image by reflection in the origin point is itself is ……………
(a) (1 , 0) (b) (0 , 1) (c) (0 , 0) (d) (– 1 , 0)

3 The image of the point (– 2 , 3) by refletion in the y-axis is the point ……………
(a) (3 , 2) (b) (– 3 , 2) (c) (2 , 3) (d) (– 3 , – 2)

4 The image of the point (– 1 , 2) by translation of magnitude of 3 units in the positive direction of the x-axis is ……………
(a) (– 1 , 5) (b) (2 , 2) (c) (– 2 , 2) (d) (– 1 , 3)

5 The image of the point (– 4 , 5) by translation (2 , – 3) is ……………
(a) (2 , 2) (b) (– 2 , 2) (c) (2 , – 2) (d) (– 2 , – 2)

6 The equivalent rotation to R (O , 90°) is R (O , ……………°)
(a) – 90 (b) 270 (c) 180 (d) – 270

7 The image of the point (– 2 , 1) by reflection in the origin point is ……………
(a) (2 , 1) (b) (– 2 , 1) (c) (2 , – 1) (d) (– 2 , – 1)

8 The image of the point …………… by reflection in y-axis is (3 , 2)
(a) (3 , – 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

9 The image of the point (1 , – 2) by translation $(x , y) \longrightarrow (x - 1 , y + 3)$ is ……………
(a) (2 , – 5) (b) (0 , – 1) (c) (0 , 1) (d) (2 , 5)

10 The image of the point (– 4 , 3) by geometric transformation $(x , y) \longrightarrow (-x , y - 7)$ is ……………
(a) (– 4 , – 4) (b) (4 , – 4) (c) (4 , 10) (d) (– 4 , 10)

11 The image of the point (– 1 , 4) by rotation R (O , 180°) is ……………
(a) (– 1 , – 4) (b) (1 , – 4) (c) (4 , 1) (d) (1 , 4)

12 The image of the point (3 , – 7) by reflection in the origin point is ……………
(a) (– 3 , 7) (b) (– 3 , – 7) (c) (3 , – 7) (d) (3 , 7)

13 If the image of the point (9 , – 4) by rotation about the origin point is itself , then the measure of the rotation angle is ……………
(a) 90° (b) 180° (c) 270° (d) 360°

14	The image of the point (– 3 , 2) by reflection in y-axis is .... (a) (– 3 , – 2) (b) (3 , – 2) (c) (2 , – 3) (d) (3 , 2)
15	The image of a rhombus by any translation is a .... **(a)** rhombus. **(b)** rectangle. **(c)** square. **(d)** trapezium.
16	The image of the point (3 , – 2) by translation (– 1 , 6) is .... (a) (2 , 4) (b) (– 2 , 4) (c) (7 , – 8) (d) (2 , – 4)
17	The image of a triangle by rotation around the origin point with an angle of measure 180° is .... (a) a triangle. (b) a line segment. (c) a point. (d) a straight line.
18	If À is the image of A by reflection in M and MA = 6 cm. , then AÀ = ... cm. (a) 6 (b) 3 (c) 12 (d) 9
19	The image of the point (5 , 1) by reflection in the origin point is .... **(a)** (1 , 5) **(b)** (– 1 , – 5) **(c)** (– 5 , – 1) **(d)** (– 1 , 5)
20	The image of the point (– 3 , 4) by reflection in the y-axis is .... (a) (3 , 4) (b) (3 , – 4) (c) (– 3 , – 4) (d) (4 , – 3)
21	The image of the square by any translation is a .... (a) rectangle (b) square (c) rhombus (d) trapezium
22	The image of the point (3 , – 2) by translation (– 1 , 4) is .... (a) (2 , 2) (b) (– 4 , – 6) (c) (2 , – 2) (d) (2 , 6)
23	The image of a triangle by rotation about the origin point with an angle of measure 360° is .... (a) point. (b) triangle. (c) line segment. (d) straight line.
24	**In the opposite figure :** Δ AB̀C̀ is the image of Δ ABC by rotation about A with angle of measure .... B̀ C̀ C B A 80° 30° 30° (a) 30° (b) 80° (c) 110° (d) 140°
25	The reflected image of the point A (– 3 , 2) in the origin point is the point À (............... , ...) (a) (3 , – 2) (b) (3 , 2) (c) (– 3 , – 2) (d) (2 , – 3)
26	The image of the point (– 3 , 2) by reflection on the x-axis is .... (a) (– 3 , – 2) (b) (3 , – 2) (c) (3 , 2) (d) (– 3 , 2)
27	The image of the point (3 , 5) by reflection in y-axis is .... (a) (– 3 , – 5) (b) (3 , – 5) (c) (– 3 , 5) **(d)** (– 5 , – 3)

[B] : Complete the Following : -

1	If $\grave{A}$ is the image of A by reflection in M and MA = 6 cm. , then A$\grave{A}$ = ... cm.
2	The image of the point (2 , – 4) by reflection in x-axis is ....
3	The image of the point (2 , – 1) by rotation about the origin point with an angle of measure 180° is ....
4	**Translation (x , y) ⟶ (x + 1 , y – 3) maps the point (3 , 4) to ....**
5	The image of the point (2 , 3) by translation MN , in direction $\overrightarrow{MN}$, where M (2 , – 1) , N (5 , 1) is ....
6	The image of the point (2 , – 4) by rotation about the origin point with an angle of measure 90° is ....
7	The image of the point .... by rotation about the origin point with an angle of measure 180° is (– 3 , – 2)
8	The image of the point (1 , 3) by reflection in the x-axis is ....
9	The image of (3 , – 4) by reflection in y-axis is ....
10	The image of the point (3 , – 2) by translation (x , y) ⟶ (x – 1 , y + 6) is ....
11	If $\grave{A}$ (7 , – 2) is the image of A by the translation whose rule is (x , y) ⟶ (x – 3 , y + 1) , then A = ...
12	The image of the point (2 , 3) by rotation R (O , 90°) is ....
13	The image of the point (4 , –1) by rotation about the origin point with an angle of measure 180° is ....
14	The image of the point (2 , 4) by reflection in y-axis is ....
15	The image of the point (2 , 5) by translation (x , y) ⟶ (x + 2 , y – 3) is ....

16	If the image of the point $(-2, 4)$ by a translation is $(2, 5)$, then the image of the point $(2, -1)$ by the same translation is ....
17	The image of the point $(2, -3)$ by rotation about the origin point with an angle of measure $90°$ is ....
18	The image of the point $(3, -8)$ by rotation about the origin point with an angles of measure $180°$ is ....
19	If $\grave{A}(-3, -3)$ is the reflected image of the point $A(x, y)$ in the origin point $(0, 0)$, then $x =$ .... and $y =$ ....
20	The image of the point $(-1, 3)$ by reflection in the y-axis is ....
21	The image of the point $(-1, 3)$ by translation $(1, 4)$ is ....
22	The image of the point $(-5, 6)$ by the translation $(-2, 3)$ is the point ....
23	The image of the point .... by rotation about the origin point with an angle of measure $90°$ is $(-1, 4)$
24	The image of the point $(3, 8)$ by rotation about the origin point with an angle of measure $180°$ is ....
25	If the image of the point A by reflection in the origin is $\grave{A}(3, -2)$, then A is (.......... , ...)
26	If the point $\grave{A}(3, -2)$ is the image of A by reflection on the x-axis, then A = (....... , ...)
27	The image of the point $(-1, 3)$ by translation $(4, -2)$ is ....
28	The image of the point $(5, -3)$ by translation 3 units in negative direction of x-axis is ....
29	The image of the point $(-1, 2)$ by rotation about the origin point with angle of measure $90°$ is ....
30	The image of $(3, 7)$ by rotation $R(O, 180°)$ is ....

[C] : Essay Problems : -

1	In a cartesian plane draw the image of Δ ABC where A $(-2, 3)$, B $(2, 3)$, C $(2, 6)$ by translation $(x, y) \longrightarrow (x+2, y-1)$ 2018 Exam (9) Question (4) (b)
2	Draw Δ ABC in which AB = 3.5 cm. , m ($\angle$ A) = 90° , AC = 5 cm. , then draw its image by reflection in $\overleftrightarrow{AC}$ 2017 Exam (9) Question (4) (a)
3	Using the lattice , draw Δ ABC where A $(1, 0)$, B $(0, 2)$ and C $(-3, 1)$, then draw its image by reflection in x-axis. 2017 Exam (1) Question (5) (b)
4	Draw the triangle ABC in which : AB = AC = 5 cm. and BC = 6 cm. , then draw its image by rotation about A with an angle of mesure 180° 2016 Exam (13) Question (4) (b)
5	Draw Δ OBC on square lattice where : O $(0, 0)$, B $(3, 0)$ and C $(0, 4)$, then find its image by rotation R (O , – 90°) 2016 Exam (10) Question (4) (a)
6	Using the lattice , draw Δ ABC in which A $(-4, 1)$, B $(-1, 3)$ and C $(0, 1)$, then draw the image of Δ ABC by translation $(2, 1)$ 2017 Exam (9) Question (3) (a)
7	Draw the triangle ABC in which AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then draw its image by reflection in $\overleftrightarrow{BC}$ 2017 Exam (8) Question (5) (b)
8	On a square lattice , draw Δ ABC where A $(1, 1)$, B $(4, 1)$, C $(4, 4)$ then draw its image by reflection in x-axis. 2017 Exam (5) Question (3) (a)
9	**In the opposite figure :** LMN is an equilateral triangle X , Y and Z are midpoints of $\overline{LM}$, $\overline{MN}$ and $\overline{NL}$ respectively. **Find :** (1) The image of Δ LXZ by reflection in $\overleftrightarrow{XZ}$ (2) The image of $\overline{XY}$ by rotation (Y , 60°) 2017 Exam (13) Question (3) (a)
10	On a square lattice , draw Δ ABC where : A $(4, 4)$, B $(4, 2)$ and C $(1, 2)$, then draw : (1) The image of Δ ABC by reflection in the y-axis. (2) The image of Δ ABC by rotation about the origin point with an angle of measure 270° 2016 Exam (11) Question (5) (a)

11	Using the lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) , then find the image of $\overline{AB}$ by translation (X , Y) ⟶ (X + 2 , Y – 1) 2018 Exam (14) Question (3) (b)
12	Draw the triangle ABC in which : AB = 7 cm. , BC = 6 cm. and CA = 8 cm. , then draw its image by reflection in $\overleftrightarrow{BC}$ 2016 Exam (10) Question (3) (b)
13	Draw Δ ABC where A (1 , 1) , B (4 , 1) , C (4 , 5) find its image by reflection in y-axis. By using square lattice. 2018 Exam (11) Question (4) (b)
14	**In the opposite figure :** ABCD is a square , M is the intersection point of its diagonal , find the image of Δ MAB by rotation about M with angle 90° D A M C B 2018 Exam (7) Question (5) (a)
15	Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) , then draw its image by rotation about the origin point with an angle of measure 180° 2018 Exam (2) Question (5) (a)
16	On a square lattice , draw Δ ABC where A (4 , 4) , B (0 , 2) , C (6 , – 2) then find its image by translation (x , y) ⟶ (x – 4 , y + 1) 2017 Exam (6) Question (3) (a)
17	**In the opposite figure :** ABCD is a parallelogram , M is the point of intersection of its diagonals and X ∈ $\overline{AC}$, Y ∈ $\overline{AC}$, such that m (∠ ABX) = m (∠ CDY) D A X M Y C B **Prove that :** ① Δ ABX is the image of Δ CDY by reflection in M ② The figure XBYD is a parallelogram. 2018 Exam (9) Question (5)
18	On a square lattice , draw the image of square ABDC where : A (1 , 2) , B (– 2 , 2) , C (1 , 5) , D (– 2 , 5) by reflection in the y-axis. 2018 Exam (15) Question (4) (a)
19	On the square lattice draw Δ ABC where A (1 , 1) , B (4 , 1) , C (4 , 4) **, then determine each of the following :** ① The image of Δ ABC by reflex in y-axis. ② The image of Δ ABC by rotation about origin point with angle of measure 180°

	2018 Exam (5) Question (5)
20	If the image of the point A (1 , 1) by translation in the coordinate plane is À (2 , 2) , find the images of the points O (0 , 0) , B (4 , 2) , C (– 3 , 5) by the same translation. 2018 Exam (7) Question (4) (b)
21	**In the opposite figure :** ABCD is a square , whose diagonals intesect at M. Find the image of Δ MAB By rotation about M with angle of measure (– 90°) 2018 Exam (14) Question (5) (b)
22	If the point A is the image of the point (– 1 , 2) by reflection in y-axis , then find the image of A by translation (– 1 , 2) 2017 Exam (2) Question (3) (a)
23	Draw the image of the square ABCD where A (2 , 4) , B (2 , 1) , C (5 , 1) , D (5 , 4) by reflection in the X-axis. 2018 Exam (7) Question (3) (b)
24	**In the opposite figure :** ABCD is a square X , Y , Z , L are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively **Find :** ① The image of Δ AML by translation with a magnitude AM in direction $\overrightarrow{AM}$ ② The image of Δ AML by rotation about the point M with an angle of measure – 90° ③ The image of Δ AML by reflection in $\overleftrightarrow{LY}$ 2018 Exam (2) Question (4) (b)
25	Using the lattice , draw Δ ABC where : A (2 , 3) , B (4 , 2) and C (1 , 2) , then draw its image by rotation about the origin point with an angle of measure 180° 2016 Exam (15) Question (3) (a)
26	**In the opposite figure :** XYZL is a trapezium , in which $\overline{XL}$ // $\overline{YZ}$, m (∠ Y) = 90° , $\overline{LM}$ ⊥ $\overline{YZ}$, YZ = ZL = 17 cm. , XL = 9 cm. **Find :** ① The length of $\overline{XY}$ ② The image of $\overline{XL}$ by translation of magnitude XY in the direction of $\overrightarrow{XY}$ 2018 Exam (3) Question (5)

No.	Question
27	In a Cartesian plane draw the image of Δ ABC Where A (1 , 0) , B (0 , 2) , C (– 3 , 1) by translation $(x , y) \longrightarrow (x + 2 , y - 1)$ 2017 Exam (8) Question (4) (b)
28	On a square lattice , draw Δ ABC where : A (4 , 4) , B (4 , 2) and C (1 , 2) , then find its image by reflection in the y-axis. 2016 Exam (14) Question (5) (b)
29	**In the opposite figure :** ABCDEF is a regular hexagon **Find the image of Δ ABM by :** ① Reflection on $\overleftrightarrow{EB}$ ② Translation FE in direction of $\overrightarrow{FE}$ ③ Rotation (M , 120°) ④ Reflection in M ⑤ Rotation (M , 300°) 2018 Exam (6) Question (4)
30	**Using the square lattice :** Draw Δ XYZ in which X (4 , 1) , Y (5 , 0) , Z (– 1 , – 2) , then draw its image by rotation about the origin point with an angle of measure (– 180°) 2018 Exam (13) Question (3) (a)
31	**Complete :** Rotation in a plane reserve .... , ... 2018 Exam (15) Question (3) (a)
32	On the square lattice , draw Δ ABC in which : A (– 2 , 2) , B (1 , 4) and C (3 , 1) , then find the image of Δ ABC by translation (2 , 1) 2016 Exam (6) Question (5) (a)
33	Draw the triangle ABC in which AB = 3 cm. , BC = 4 cm. , m ($\angle$ B) = 90° , then draw its image by reflection in straight line $\overleftrightarrow{BC}$ 2018 Exam (8) Question (5) (b)
34	In a square lattice , draw the triangle OBC in which O (0 , 0) , B (3 , 0) , C (1 , 2) then find its image by reflection in the y-axis. 2017 Exam (14) Question (5) (a)
35	A rectangle its vertices are A (– 1 , – 2) , B (7 , 2) , C (5 , 6) and D (– 3 , 2) Find the image of the vertices by rotation around the origin point with an angle of measure 180° 2017 Exam (4) Question (5) (a)